MA 15200 Lesson 13 Section 1.4

I The imaginary unit *i*

Even though it does not seem logical to speak of square roots of negative numbers, they are important and play a role in some algebra and science fields.

A number i is defined to be the solution of the equation $x^2 = -1$. Therefore ...

The **imaginary unit *i*** is defined by $i = \sqrt{-1}$, where $i^2 = -1$.

A square root of a negative number can be represented using i in the following manner.

$\sqrt{-36} = \sqrt{36} \cdot \sqrt{-1} = 6i$

<u>Ex 1:</u> Write each number using the imaginary unit.

$a)\ \sqrt{-100}$

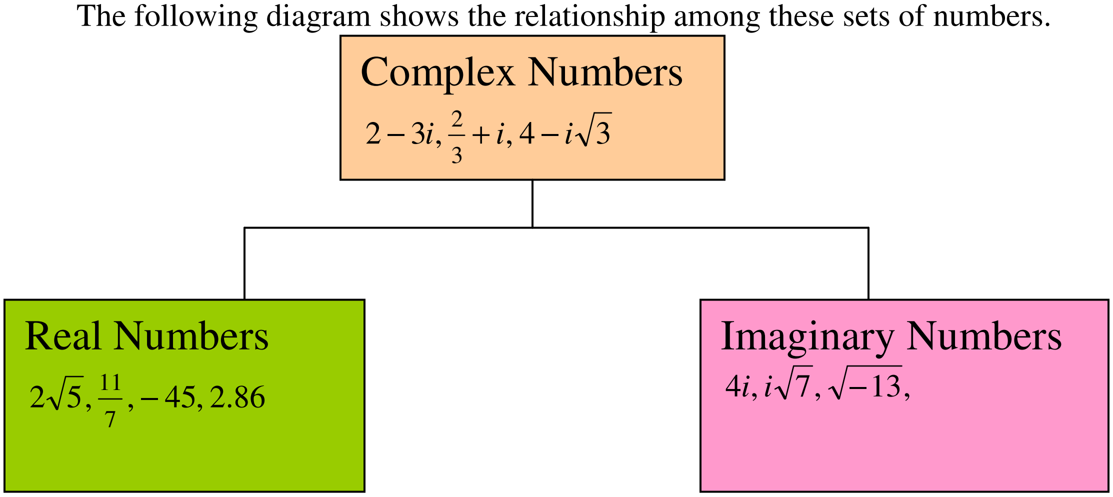
$b)\ \sqrt{-\frac{49}{4}}$

$c)\ \sqrt{-18}$

II The Set of Complex Numbers

A new set of numbers is created using imaginary numbers. A **complex number** is formed by adding a real number to an imaginary number.

The following diagram shows the relationship among these sets of numbers.



A complex number is written in $a + bi$ form (standard form), where a is the 'real part' and bi is the 'imaginary part'.

If $a = 0$ $(0+ bi)$, the number is a pure imaginary number.

If $b = 0$, $(a + 0\text{i})$, the number is a pure real number.

Therefore, every real number can be written as a complex number and every imaginary number can be written as a complex number.

If b is contains a radical, we usually write the i before the radical. These are examples of complex numbers: $2-3i,\ \ 4+\frac{2}{3}i,\ \ -8,\ \ 5i,\ \ 6+i\sqrt{2},\ \ \pi-3i\sqrt{5}$

III Operations with Complex Numbers

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$
$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

Basically, combine like terms.

Ex 2: Add or subtract and write in standard form.

$a)\quad (6-2i)+(3+14i)=$

$b)\quad \left(\frac{3}{4}-3i\right)-\left(\frac{2}{3}+7i\right)=$

$c)\quad (14-\sqrt{-16})-(\sqrt{-81}-2)=$

$d)\quad (3i+12)+(17-\sqrt{-49})=$

$e)\quad (4-2i)-(6+3i)+(12-2i)=$

Note: You must change the square roots of negative numbers to pure imaginary number before adding or subtracting.

IV Multiplication of Complex Numbers

$$(a+bi)(c+di)=ac+adi+bci+bdi^2$$
$$=ac+adi+bci-bd\quad (i^2=-1)$$
$$=(ac-bd)+(ad+bc)i$$

Basically, use FOIL when multiplying and remember to simplify $i^2=-1$ and combine 'like' terms.

Ex 3: Find each product in standard form.

$a)$ $(4-5i)(2-6i)=$

$b)$ $(4+\sqrt{-9})(-2-\sqrt{-36})=$

$c)$ $(4-7i)^2=$

$d)$ $(6-i\sqrt{2})(6+i\sqrt{2})$

$e)$ $4i\left(\frac{3}{4}-\frac{1}{4}i\right)$

V Complex Conjugates and Division of Complex Numbers

Remember the conjugate of $a+bi$ is $a-bi$ and vice-versa. The product of conjugates is a^2-b^2. Because the square of a real number is always a rational number, the product of two conjugates will be rational (without a radical). This product will help us divide complex numbers. Multiply numerator and denominator by the conjugate of the denominator, just like you did when rationalizing a fraction with a binomial denominator containing a radical.

Ex 4: Divide and write answer in standard form.

$a)$ $\frac{12}{3i}=$

$b)\quad \dfrac{-3}{2-4i} =$

$c)\quad \dfrac{4+2i}{3-i} =$

$d)\quad \dfrac{12+\sqrt{-9}}{-3-\sqrt{-16}}$

Ex 5: Perform the operations and write in standard form.

$a)\quad \sqrt{-121}-\sqrt{-196} =$

$b)\quad \left(\sqrt{-100}\right)\left(\sqrt{-81}\right) =$

$c)\quad (2-\sqrt{-7})^2$

$d)\quad \dfrac{-15-\sqrt{-75}}{10}$

Ex 6: Complex numbers are used in electronics to describe the current in an electric circuit. Ohm's law relates the current, *I*, in amperes, the voltage, *E*, in volts and the resistance, *R*, in ohms by the formula $E = IR$. If the voltage is 6 - 2*i* volts and the resistance is 5 + *i* ohms, find the current.

VI (Optional) Powers of the Imaginary Unit

There is a pattern in the powers of the imaginary unit, *i*.

$i^0 = 1$	$i^4 = i^2 i^2 = (-1)(-1) = 1$	$i^8 = 1$
$i^1 = i$	$i^5 = i^4 i = 1i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 i^2 = (1)(-1) = -1$	$i^{10} = -1$
$i^3 = i^2 i = -1i = -i$	$i^7 = i^4 i^3 = 1(-i) = -i$	$i^{11} = -i$

Notice: Even powers of *i* are either 1 or -1 and odd powers of *i* are either *i* or -*i*.

To evaluate a power of *i*: i^n

1. Determine how many groups of 4 are in *n* by dividing *n* by 4.
2. The power can be written as $i^n = i^r$ where *r* is the remainder after dividing.
3. Simplify i^r. The answer will be either 1, -1, *i*, or -*i*.

<u>Ex 4:</u> Evaluate each power.

$a)$ $i^{49} =$

$b)$ $i^{256} =$