Lesson 30   Section 6.1
Rational Expressions and Functions

A **Rational Expression** is a polynomial divided by a non-zero polynomial.

The following are examples of rational expressions.

$$\frac{3}{4}, \quad \frac{2w}{x}, \quad \frac{9+x}{5}, \quad \frac{r^2-r}{3r+1}, \quad \frac{y^2-2y+5}{y^2-8}$$

A **Rational Function** is a function where $f(x)$ is represented by a rational expression.

1)   Evaluate the following rational function for the given values.

$$G(x) = \frac{3x^2 - x + 1}{x - 5} \text{ for } x = 0, \text{-2, 3, and 5}$$

Remember: No fraction (rational expression can have a zero denominator. When a rational expression or rational function has a single variable, the set of numbers that could be replaced for that variable is called the **domain.** No number can be included in the domain that makes a zero denominator when replaced for **x** (or the variable). When these numbers are determined, it is said these are the **domain restrictions**.

For each rational expression or rational function, list all domain restrictions.

A $r(x) = \dfrac{4+x}{3x} \qquad x \neq$

B $f(x) = \dfrac{2x}{(x+3)(x-5)} \qquad x \neq$

C $g(x) = \dfrac{2}{x^2(x+4)(x-4)} \qquad x \neq$

Procedure for **Simplifying Rational Expression:**
  I.   Factor each denominator and numerator.
  II. You may cancel the same **factor** found in a numerator and a denominator, since
       they equal a factor of 1.
            You may also cancel a factor in a numerator that is opposite of a factor in the
            denominator and that quotient equals a factor of -1.
  III. The simplified rational expression is the factors that remain.
Caution: *Cancel only Factors, never Terms!!!!*

Simplify each expression. Determine any domain restrictions.

2) $\dfrac{25}{30}$

3) $\dfrac{(x-2)(x+3)}{(x+3)(2x-5)}$

4) $\dfrac{4a^2b^3}{6a^3b}$

5) $\dfrac{3a+12}{6a}$

6) $f(x) = \dfrac{y^2-y-6}{3y^2+6y}$

7) $\dfrac{4y-20}{4y^2+12y}$

8) $h(x) = \dfrac{4x^2 - 9}{6x^2 + 9x}$

Remember: Opposite reduce to -1 $\quad \dfrac{-3}{3} = -1 \qquad \dfrac{x}{-x} = -1 \qquad \dfrac{b-a}{a-b} = -1$

9) $\dfrac{(2-x)(3+x)}{x(x-2)}$

Procedure for **Multiplying or Dividing Rational Expressions** and writing the product or quotient in simplified form:
  I.   Factor each numerator and denominator.
  II. Cancel any factor the same in the numerator and denominator or opposite factors.
  III. Multiply across and leave in factored form.
  IV. With division, multiply by the reciprocal of the divisor.

Multiply or divide. Write answers in simplified form.

10) $\dfrac{16a^2}{3b^2} \div \dfrac{8a^3}{6b}$

11) $\dfrac{5m^2}{4m-8} \bullet \dfrac{6m-12}{10m}$

12) $\dfrac{a^2-1}{2-5a} \cdot \dfrac{15a-6}{a^2+5a-6}$

13) $\dfrac{x^2-y^2}{4x+4y} \div \dfrac{3y-3x}{12x^2}$

14) $\dfrac{y^2-36}{y^2-8y+16} \div \dfrac{3y-18}{y^2-y-12}$

15) $\dfrac{2x^2+9x+9}{x^2+3x} \div \dfrac{4x^2-9}{2x^3-3x^2}$