MA 15200 Lesson 28 Section 4.2

Remember the following information about inverse functions.

1. In order for a function to have an inverse, it must be one-to-one and pass a horizontal line test.
2. The inverse function can be found by interchanging *x* and *y* in the function's equation and solving for *y*.
3. If $f(a)=b,$ then $f^{-1}(b)=a$. The domain of *f* is the range of f^{-1} and the range of *f* is the domain of f^{-1}.
4. The compositions $f(f^{-1}(x))$ and $f^{-1}(f(x))$ both equal *x*.
5. The graph of f^{-1} is the reflection of the graph of *f* about the line $y=x$.

Because an exponential function is 1-1 and passes the horizontal line test, it has an inverse. This inverse is called a logarithmic function.

I Logarithmic Functions

According to point 2 above, we interchange the *x* and *y* and solve for *y* to find the equation of an inverse function.

$f(x)=b^x$ exponential function

$x=b^y$ inverse function How do we solve for *y*? There is no way to do this. Therefore a new notation needs to be used to represent an inverse of an exponential function, the logarithmic function.

> Definition of Logarithmic Function
> For $x>0$ and $b>0$ $(b\neq 1)$
>
> $y=\log_b x$ is equivalent to $x=b^y$
>
> The function $f(x)=\log_b x$ is the **logarithmic function with base *b*.**

The equation $y=\log_b x$ is called the logarithmic form and the equation $x=b^y$ is called the exponential form. The value of *y* in either form is called a **logarithm.** Note: The logarithm is an exponent.

Exponential Form

$x=b^y$

argument (x) base (b) exponent (y)

Logarithmic Form

$y=\log_b x$

exponent (y) base (b) argument (x)

Ex 1: Convert each exponential form to logarithmic form and each logarithmic form to exponential form.

$a)\quad 3^4 = 81$

$h)\quad m^{rp} = x + 4$

$b)\quad \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

$i)\quad (2a)^{y+7} = p^2$

$c)\quad 25^{\frac{1}{2}} = 5$

$d)\quad 8^{-2} = \frac{1}{64}$

$e)\quad \log_2 32 = 5$

$j)\quad \log_q (2mn) = 12$

$f)\quad \log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$

$g)\quad \log_5 \sqrt{5} = \frac{1}{2}$

$k)\quad \log_{x+3} 200 = rs$

II Finding logarithms

Remember: A logarithm is an exponent.

Ex 2: Find each logarithm.

$a)\quad \log_{10} 100{,}000$

$b)\quad \log_3 27$

$c)\quad \log_{20} 1$

$d)\quad \log_{15} 15$

$e)\quad \log_{12} \frac{1}{144}$

$f)$ $\log_4 64$

$g)$ $\log_{\frac{1}{2}} 32$

$h)$ $\log_3 81$

III Basic Logarithmic Properties

1. $\log_b b = 1$ Since the first power of any base equals that base, this is reasonable.
2. $\log_b 1 = 0$ Since any base to the zero power is 1, this is reasonable.

The exponential function $f(x) = b^x$ or $y = b^x$ and the logarithmic function $f^{-1}(x) = \log_b x$ or $y = \log_b x$ are inverses.
$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

This leads to 2 more basic logarithmic properties.

3. $\log_b b^x = x$ This is a composition function where
$f(x) = b^x$ and $f^{-1}(x) = \log_b x$. $f^{-1}(f(x)) = f^{-1}(b^x) = \log_b b^x = x$ (the exponent)
4. $b^{\log_b x} = x$ This is a composition function where
$f(x) = b^x$ and $f^{-1}(x) = \log_b x$. $f(f^{-1}(x)) = f(\log_b x) = b^{\log_b x} = x$ (the number or argument)

<u>Ex 3:</u> Simplify using the basic properties of logarithms.

$a)$ $\log_4 1 =$

$b)$ $\log_3 3 =$

$c)$ $12^{\log_{12} 4} =$

$d)$ $\log_{10} 10^5 =$

Ex 4: Simplify, if possible.

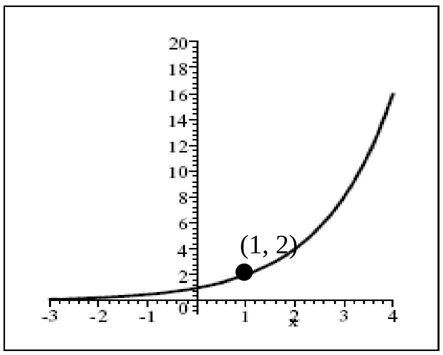
$a)$ $\log_{(-4)} 1 =$

$b)$ $\log(-100) =$

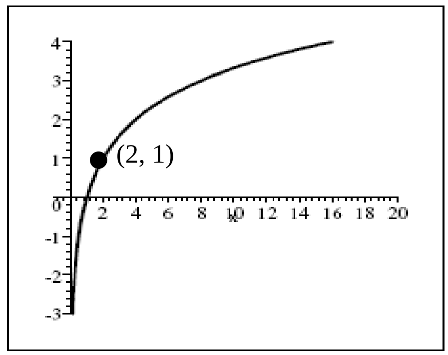
Remember that bases must be positive and the argument values (the numbers) must be positive.

IV Graphs of Logarithmic Functions

Below is a graph of $y = 2^x$ and its inverse, $x = 2^y$.

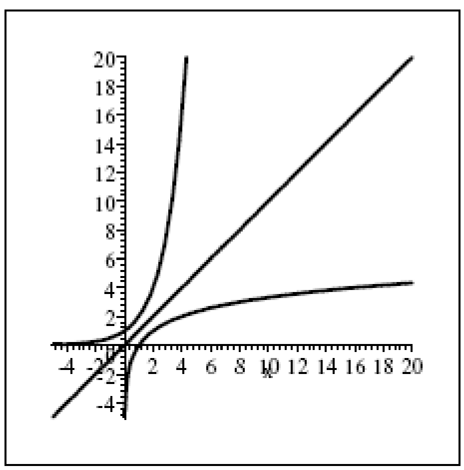


$y = 2^x$



$x = 2^y$

If you imagine the line $y = x$, you can see the symmetry about that line.
Below are both graphs on the same coordinate system along with $y = x$.



Characteristics of a logarithmic Graph:

The **inverse of this function**, $x = b^y$, has a graph with the following characteristics.

1. The ***x*-intercept** is (1, 0).
2. The graph still is increasing if $b > 1$, decreasing if $0 < b < 1$.
3. The **domain** is all positive numbers, so the graph is to the right of the *y*-axis. (The range is all real numbers.)
4. The ***y*-axis is an asymptote.**

V Common Logarithms

A logarithmic function with base 10 is called the **common logarithmic function.** Such a function is usually written without the 10 as the base.

$\log x$ is equivalent to $\log_{10} x$

A calculator with a key [log] can approximate common logarithms.

Put the number (argument) in the calculator, press the common log key.

Ex 5: Find each common logarithm **without a calculator**.

a) $\log 1000 =$

b) $\log \dfrac{1}{100} =$

c) $\log 0.001 =$

Ex 6: Use a calculator to approximate each common logarithm. Round to 4 decimal places.

a) $\log 0.025$

b) $\log 43.8$

Using the basic properties with base 10, we get the following properties.

1. $\log 10 = 1$
2. $\log 1 = 0$
3. $\log 10^x = x$
4. $10^{\log x} = x$

VI Natural Logarithms

A logarithm function with base *e* is called the **natural logarithmic function.** Such a function is usually written using ln symbol rather than log symbol and no base shown. The symbol ln means natural logarithm.

$\ln x$ is equivalent to $\log_e x$

A calculator with a key [ln] can approximate natural logarithms.

Put the number (argument) in the calculator, press the natural log key.

Ex 7: Use a calculator to approximate each natural logarithm. Round to 4 decimal places.

a) $\ln 0.988$

b) $\ln 2008$

Using the basic properties with base *e*, we get the following properties.

1. $\ln e = 1$
2. $\ln 1 = 0$
3. $\ln e^x = x$
4. $e^{\ln x} = x$

VII Modeling with logarithmic functions

The function $f(x) = 29 + 48.8\log(x+1)$ gives the percentage of adult height attained by a boy who is *x* years old.

Ex 8: Approximately what percentage of his adult height has a boy of age 11 acheived? (Notice: This model uses a common log.) Round to the nearest tenth of a percent.

The function $f(x) = 13.4\ln x - 11.6$ models the temperature increase in degrees Fahrenheit after *x* minutes in an enclosed vehicle when the outside temperature is from 72° to 96°.

Ex 9: Use the function above to approximate the temperature increase after 45 minutes. Round to the nearest tenth of a degree.