Lesson 4

Factoring:

- finding a product that is equivalent to some original algebraic or polynomial expression
- think of factoring as undoing polynomial multiplication
- not all polynomials are factorable using rational coefficients

First Factoring Method to <u>always</u> try: Take out a GCF (greatest common factor)

Greatest Common Factor (GCF):

- the largest factor that is common to each term of an expression
- the GCF of an expression could be a number, a variable, a 'grouping', or a product of these.

Example 1: Factor the following polynomials by taking out the GCF

a. $x^2 + x$ b. $17x^3y + 51xy^2$ c. $2(x+1)^2 - 3(x+1)$

Next, we will look at a polynomial with four terms.

Second factoring method to try (if the polynomial has 4 terms): The Grouping Method

$$2ax - 6bx + ay - 3by$$

We can see there is no common factor among the four (no GCF). However the first two terms do have a common factor of $2x$ and the last two terms also have a common factor (y). So while we can't factor the polynomial by taking out a GCF, we can **factor by grouping**.

$$2ax - 6bx + ay - 3by$$
$$\underline{2x(a-3b)} + \underline{y(a-3b)}$$

The two parentheses (GCF) must 'match' when using this method. Otherwise, the polynomial is probably prime (not factorable).

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We now have a sum of two terms, and both terms have a common factor of $(a-3b)$. If we take out the GCF of $(a-3b)$ we are left with the following:

$$(a-3b)(2x+y)$$

Factor by grouping:

- grouping two terms of a polynomial and factoring out a GCF from each group
- **ALWAYS check for a GCF first**

Hint: Sometimes it is helpful to 'rearrange' the terms before grouping; so the GCF of first 2 terms is not a 1.

Example 2: Factor the following polynomials

a. $x^2+4x+6xy+24y$ b. $2ay^2-axy+6xy-3x^2$

Factoring trinomials (form ax^2+bx+c) using the ac-method (product-sum method):

1. check for a GCF
2. find two numbers whose product is ac and whose sum is b
3. replace the middle term of the original polynomial (bx) with a sum expression containing the two numbers from step 2
4. factor by grouping

Third factoring method: Factoring trinomials by either 'trial-and-error' or the ***ac*** **or product-sum method**

ALWAYS check for a GCF first

Example 3: Factor the following polynomials using the ac-method (This method is sometimes called the product-sum method).

a. $12x^2-x-6$ b. $4x^3+4x^2+x$

c. $18 - 15x - 3x^2$
$= -3(\qquad\qquad\qquad)$

A 'short-cut' method can be used when the leading coefficient is a 1.

d. $9x^2 + 30x + 25$

ac = 225, *b*=30

e. $8x^3 - 22x^2 + 15x$

ac = 120, *b* = -22

f) $x^2 - 3x - 10$

ac= -10, *b* = -3

There is another way to factor the trinomial $9x^2 + 30x + 25$. Notice that $9x^2$ is $(3x)^2$, 25 is $(5)^2$, and $30x$ is $2(3x)(5)$.

$$9x^2 + 30x + 25$$
$$(3x)^2 + 2(3x)(5) + (5)^2$$
$$(3x + 5)(3x + 5)$$
$$(3x + 5)^2$$

This is an example of factoring a **Perfect Square Trinomial**. There is a pattern to recognize how to factor a perfect square trinomial.

Pattern for a perfect square trinomial:
$a^2 \pm 2ab + b^2 = a^2 \pm 2ab + b^2 = (a \pm b)^2$

4th factoring method: Perfect Square Trinomial Pattern

<u>Perfect Square Trinomials:</u>

- trinomials with first and last terms that are perfect squares **<u>AND</u>** can be factored using the following **<u>formulas (patterns)</u>**:
  - $x^2 + 2xy + y^2 = (x + y)^2$
  - $x^2 - 2xy + y^2 = (x - y)^2$
- **<u>ALWAYS check for a GCF first</u>**
  - $x^2 + 6x + 9 = (x)^2 + 2(x)(3) + 3^2 =$
  - $12x^3 - 72x^2 + 108x =$
  - $25a^2 + 20a + 4 =$

<u>Keep in mind that not all trinomials with first and last terms that are perfect squares are Perfect Square Trinomials</u>

(for example $x^2 - 5x + 4$).

$x^2 - 5x + 4 =$

There are three other factoring formulas. These **<u>last three formulas</u>** are used specifically to factor polynomials with only two terms (binomials).

<u>Difference of Squares:</u>

- a binomial with both terms perfect squares; one factor addition, the other subtraction
  - $x^2 - y^2 = (x + y)(x - y)$
- **<u>ALWAYS check for a GCF first</u>**
  - $x^2 - 1 =$
  - $50w^2 - 8 =$

5th factoring method: Difference of Squares Pattern

<u>Sum of Cubes/Difference of Cubes:</u>

6th factoring method: Sum or Difference of Cubes

- a binomial with both terms being perfect cubes
  - $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
  - $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- For the above pattern: The second trinomial factor can be derived from the first factor; (square 1st term , product of 2 terms , square 2nd term)
- Signs of the 2 factors can be remembered from the acronym **SOAP** (same, opposite, always positive)
- **<u>ALWAYS check for a GCF first</u>**
  - $x^3 - 1 =$
  - $y^3 + 8 =$
  - $64a^3 - 27 =$
  - $250 + 16n^6 =$

<u>THE CUBE FORMULAS WILL BE PROVIDED ON EXAMS, IF NEEDED.</u>

<u>When factoring polynomials with real coefficients, there is no sum of squares factoring pattern.</u>

A sum of square is always 'prime', unless there is a GCF.

<u>My steps for factoring:</u>

1. **<u>ALWAYS check for a GCF first</u>**
2. If the polynomial has two terms (binomial), check to see if both terms are perfect squares or perfect cubes.
   a. If possible use the formulas
      i. $x^2 - y^2 = (x + y)(x - y)$
      ii. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
      iii. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
   b. If not possible, the polynomial is not factorable.

The 'difference of squares' pattern (formula) may be used more than once when factoring.

If a GCF has been taken out first, the trinomial factor from the cubic formulas will always be prime.

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3. If the polynomial has three terms, use the ac-method (product-sum method).
4. If the polynomial has four terms, try factoring by grouping.

Regardless of how you factor, always check to see if any factors are still factorable (factor until you cannot factor any more, factor completely).

- $x^4 - 1 =$

Example 4: Factor **completely**.

a. $x^4 - 16x^2$

b. $25z^2 - 30z + 9$

c. $5x^3 + 10x^2 - 20x - 40$

d. $12x^2 - 29x + 15$

e. $x^6 - 81x^2$

f. $64x^3 - y^6$

g. $2x^4 + 250x$

h. $x^6 + 7x^3 - 8$