MA 15300 Exam 3B Fall 2012

Name: ____ Purdue ID: ________________________

Instructions:

1. Fill in your name and Purdue ID above.
2. It is suggested you use a pencil, not an ink pen.
3. There are 15 questions on the exam. Do all of your work on the question sheet. Circle the capital letter in front of your answer choice.
4. All questions are worth the same. **Please answer every question**. There is no penalty for guessing.
5. A TI-30XA scientific calculator is the ONLY calculator that may be used on the exam. **No other calculators are allowed. Cell phones, iPods, books, and scrap paper are also NOT allowed.**
6. The exam is self-explanatory. Do **NOT** ask any questions about any of the exam problems unless you believe there is a printing or typing error.
7. When you are finished, give your exam to your instructor. You may leave then.

Circle your section number.

8:40 – 9:40 001 9:50 – 10:50 002

Use the functions, $f(x)=\dfrac{x+8}{x-7}$ and $g(x)=x^2-9$ to answer questions #1 and #2:

1. Find $(f-g)(6)$.

$= f(6)-g(6)$

$= \dfrac{6+8}{6-7} = (6^2-9)$

$= -14-27$

$= -41$

A. 13

B. -11

C. -13

D. -41

E. None of the above.

2. Find x, such that $(f\circ g)(x)=0$.

$f(g(x))=0$

$f(x^2-9)=0$

$\dfrac{(x^2-9)+8}{(x^2-9)-7}=0$ Denominator cannot equal zero, only the numerator.

$x^2-1=0$

$(x+1)(x-1)=0$

$x+1=0$ or $x-1=0$

$x=-1$ $\quad x=1$

A. $x=-1,1$

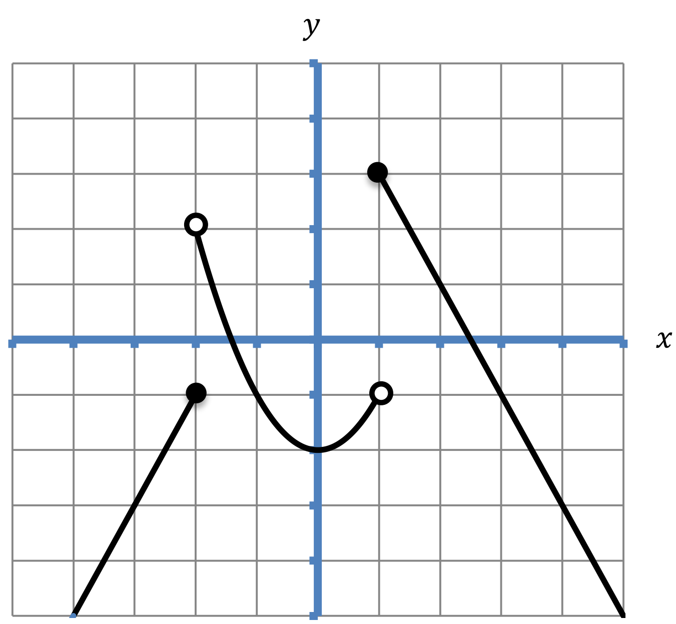
B. $x=-3,3$

C. $x=-4,4$

D. $x=7$

E. $(f\circ g)(x)\neq 0$

3. Which of the following statements about the graph of f is/are true?



I.	Increasing intervals: $(-\infty,-2]\cup[0,1)$
II.	Domain: $(-\infty,3]$
III.	x-intecept: $(0,-2)$

A. I only

B. III only

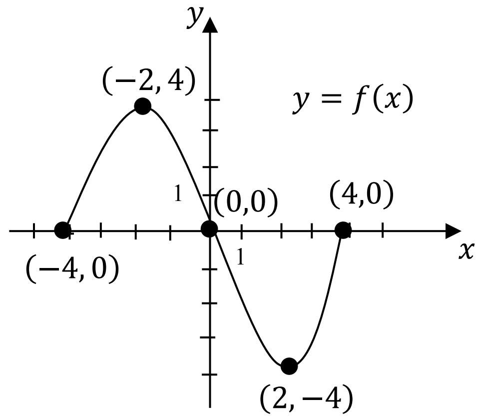
C. I and II only

D. I and III only

E. I, II, and III

I is true. It is 'rising' on those intervals
II is not true. The domain is $(-\infty,\infty)$.
III is not true. There are two x-intercepts; one between -1 and -2; the other between 2 and 3.
(The y-intercept is (0, -2).)

Use the following graph of $y = f(x)$ to answer questions #4 and #5:



4. For which of the following interval(s) is $f(x) < 0$?

The function values are negative (below x-intercept) on the interval (0, 4).

A. $(-4, 0)$

B. $(-2, 2)$

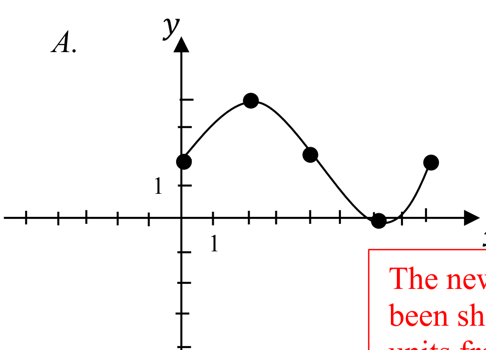
C. $(-4, -2) \cup (2, 4)$

D. $(0, 4)$

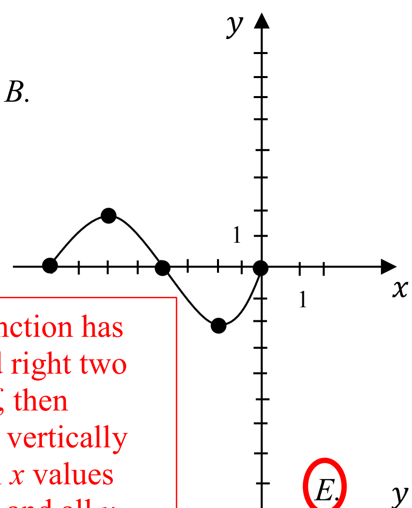
E. No such interval exists.

5. Choose the graph below that depicts $y = \frac{1}{2} f(x-4)$ if the graph $y = f(x)$ given above is the basic graph.

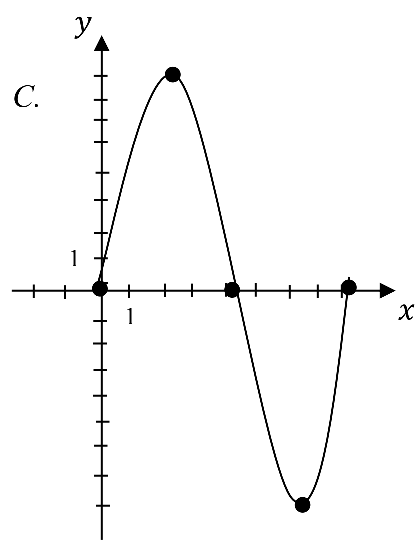
A.



B.

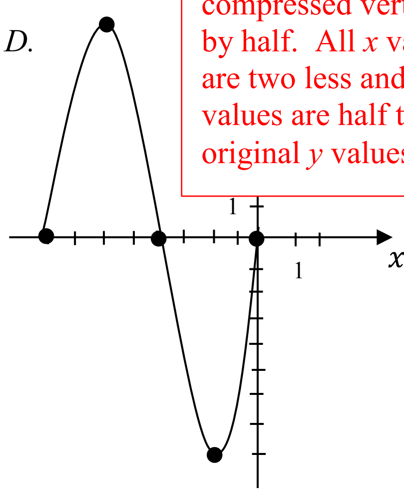


C.

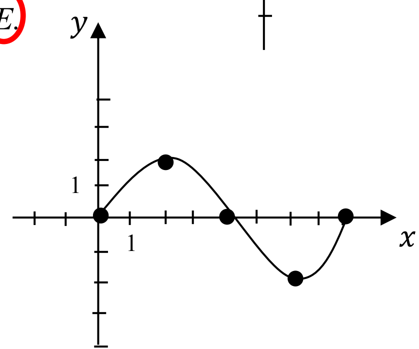


The new function has been shifted right two units from f, then compressed vertically by half. All x values are two less and all y values are half the original y values.

D.



E.



6. Solve the following system of equations. Choose the answer that describes the solution(s).

$$\begin{cases} 3x - y = 4 \\ 9x - 3y = 7 \end{cases}$$

$$\begin{cases} -3(3x - y = 4) \\ 9x - 3y = 7 \end{cases} \rightarrow \begin{cases} -9x + 3y = -12 \\ 9x - 3y = 7 \end{cases}$$

$$0 = -5$$

A false statement after eliminator indicates there is no solution.

A. The solution is $(0,0)$.

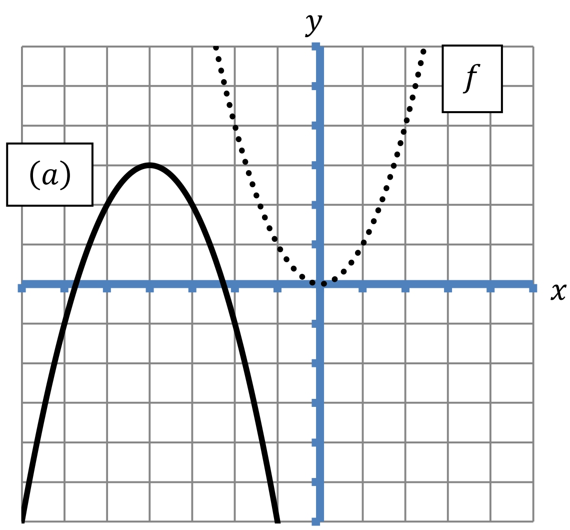
B. There is one solution. It is in Quadrant I.

C. There is no solution.

D. The solutions are ordered pairs of the form $(x, 3x-4)$.

E. None of the above.

7. The graph of a function f is shown below, together with the graph of another function (a). Use properties of symmetry, shifts, and reflecting to find the equation for the graph (a) in terms of f.



A. $y = f(-x+4) - 3$

B. $y = -f(x-4) + 3$

C. $y = f(-x-4) + 3$

D. $y = -f(x+4) + 3$

E. None of the above.

The new function (a) is a transformation of f by first shifting 4 units left, then vertically reflecting over the x-axis, and then shifting up 3 units.

8. The height $s(t)$ (in feet) of an object projected vertically upward after t seconds is given by $s(t) = -16t^2 + 80t$. Which of the following intervals represents the times when the height of the object is <u>greater than</u> 64 feet?

height $s > 64$

$-16t^2 + 80t > 64$

$-16t^2 + 80t - 64 > 0$

$-16(t^2 - 5t + 4) > 0$

$-16(t-1)(t-4) > 0$ Zeros are 1 and 4. Make a sign chart. $t \neq$ negative.

	(0,1)	(1,4)	(4,?)
$t-1$	−	+	+
-16	−	−	−
$t-4$	−	−	+
result	−	+	−

A. $(0,5)$

B. $(1,3)$

C. $(1,4)$

D. $(2,3)$

E. $(2,4)$

9. Solve the following inequality. Express your answer in interval notation.

$\dfrac{x-4}{x^2-x-20} \le 0$ Factor where possible.

$\dfrac{(x-4)}{(x-5)(x+4)} \le 0$ Zeros are 4, 5, and -4. However, only 4 could be zero.

	$(-\infty,-4)$	$(-4,4]$	$[4,5)$	$(5,\infty)$
$x-4$	$-$	$-$	$+$	$+$
$x-5$	$-$	$-$	$-$	$+$
$x+4$	$-$	$+$	$+$	$+$
result	$-$	$+$	$-$	$+$

The inequality symbol is $\le$. Select the negative and zero results.

solution: $(-\infty,-4)\cup[4,5)$

A. $[-4,4]\cup[5,\infty)$

B. $(-\infty,-4)\cup[4,5)$

C. $(-4,4]\cup(5,\infty)$

D. $[4,5)$

E. $(-\infty,-4]\cup[4,5]$

10. Which of the following is the graph of $f(x) = -x^2 - 2x - 3$?

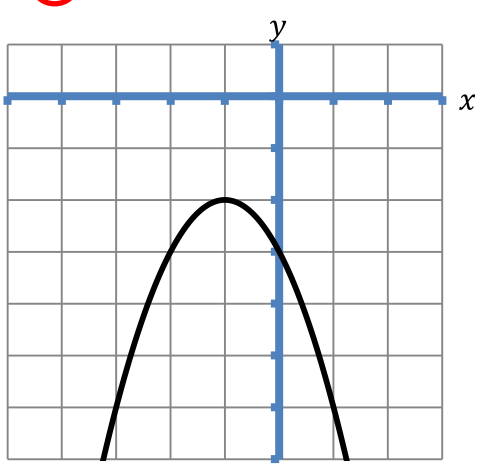
a is negative so the parabola opens down.
Find the vertex.

$h = \dfrac{-b}{2a} = \dfrac{-(-2)}{2(-1)} = -1$

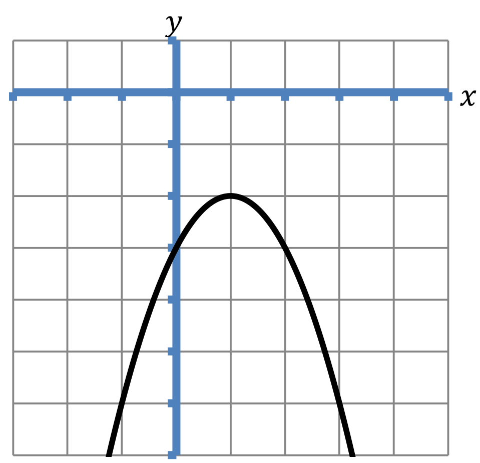
$k = f(-1) = -(-1)^2 - 2(-1) - 3 = -2$

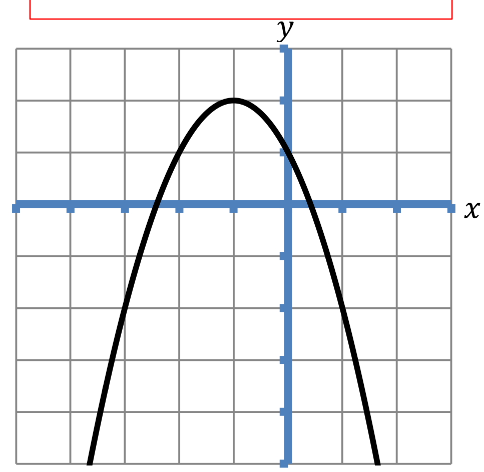
$V(-1,-2)$

A.

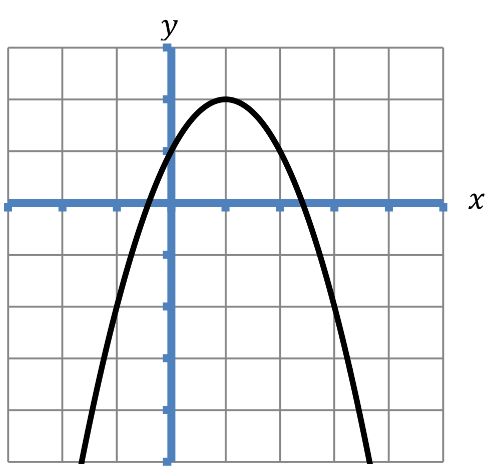


B.

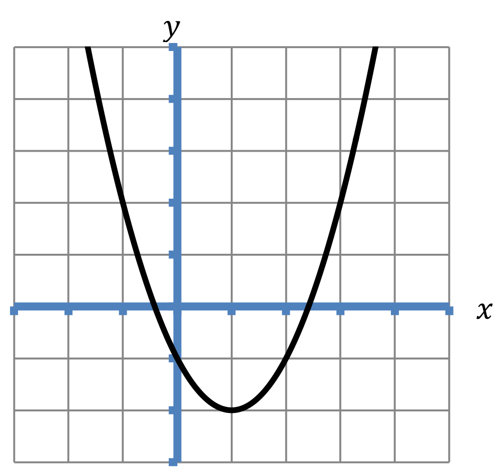




D.



E.



11. Which of the following statements is/are true of the parabola given by $f(x) = 2x^2 + 4x - 30$?

I. The zeros of f are given by $x = 5, x = -3$.

II. The minimum value of $f(x)$ is -32.

III. The standard form is $f(x) = 2(x+1)^2 - 24$.

Zeros:

$0 = 2x^2 + 4x - 30$

$0 = 2(x^2 + 2x - 15)$

$0 = 2(x+5)(x-3)$

$x + 5 = 0 \quad or \quad x - 3 = 0$

$x = -5 \qquad x = 3$

I is false.

$h = \frac{-b}{2a} = \frac{-4}{2(2)} = -1 \qquad k = f(-1) = 2(-1)^2 + 4(-1) - 30$

$k = 2 - 4 - 30 = -32$

II is true.

A. I only.

B. II only.

C. I and II only.

D. I and III only.

E. I, II, and III.

$a = 2 \quad h = -1 \quad k = -32$

$y = a(x-h)^2 + k$

$y = 2(x+1)^2 - 32$

III is false.

12. Solve the following system of equations for x:

$$\begin{cases} x + y = 10 \\ xy = 21 \end{cases}$$

Solve first equation for y and make substitution in 2nd equation.

$y = 10 - x$

$x(10 - x) = 21$

$10x - x^2 = 21$

$0 = x^2 - 10x + 21$

$0 = (x-7)(x-3)$

$x - 7 = 0 \qquad x - 3 = 0$

$x = 7 \qquad x = 3$

A. $x = -3, \quad x = -7$

B. $x = 0, \quad x = 10$

C. $x = 0, \quad x = -10$

D. $x = 3, \quad x = 7$

E. None of the above.

13. A golf ball is hit off the ground and projected upward. Its height above the ground $h(t)$ (in feet) t seconds after it is hit is given by the function $h(t) = -16t^2 + 112t$. Find the **time (in seconds)** when the ball reaches its maximum height above the ground. Which choice describes this time?

The maximum height of the ball will occur at the vertex of the quadratic function (parabola). Find the vertex.

$$h = \frac{-b}{2a} = \frac{-112}{2(-16)} = \frac{112}{32} = \frac{7}{2}$$

h represents the time.

The ball reached the maximum height in 3.5 seconds.

A. t is between 1 and 2 seconds

B. t is between 2 and 3 seconds

C. t is between 3 and 4 seconds

D. t is between 4 and 5 seconds

E. t is between 5 and 6 seconds

14. The kinetic energy E of a moving object is directly proportional to the product of the object's mass m and the square of its speed v. A rock with mass 10 kg that is moving at a rate of $6\,\frac{m}{s}$ has a kinetic energy of 180 J (joules). Determine the value of k, the constant of variation.

Variation format: $E = kmv^2$
Substitute and solve for k: $180 = k(10)(6^2)$
$180 = 360k$
$\frac{180}{360} = k \quad \frac{1}{2} = k$

A. $k = 3$

B. $k = \frac{18}{\sqrt{6}}$

C. $k = \frac{1}{3}$

D. $k = \frac{1}{2}$

E. None of the above.

15. A certain county taxes the first \$40,000 of an individual's income at 5%, and all income over \$40,000 is taxed at 8%. Find a piecewise-defined function that expresses the total tax, T, as a function of income, x. Simplify the function.

1st piece: Tax is 5% of x for $0 < x \le 40000$.
$T = 0.05x$
2nd piece: Tax is 5% of the first \$40000 plus 8% of all income over \$40000.
$T = 0.05(40000) + 0.08(x - 40000)$
$= 2000 + 0.08x - 3200$
$= 0.08x - 1200$

A. $T(x) = \begin{cases} .05x & \text{if } 0 < x \le 40{,}000 \\ .08x & \text{if } x > 40{,}000 \end{cases}$

B. $T(x) = \begin{cases} .05x & \text{if } 0 < x \le 40{,}000 \\ .08x - 1{,}200 & \text{if } x > 40{,}000 \end{cases}$

C. $T(x) = \begin{cases} .05x & \text{if } 0 < x \le 40{,}000 \\ .08x + 2{,}000 & \text{if } x > 40{,}000 \end{cases}$

D. $T(x) = \begin{cases} .05x & \text{if } 0 < x \le 40{,}000 \\ .08x - 3{,}200 & \text{if } x > 40{,}000 \end{cases}$

E. Cannot be determined.