Correct and Incorrect Notation for Limits

Example 1:

Find this limit: $\displaystyle\lim_{a \to 9} \frac{\sqrt{a} - 3}{a - 9}$

Correct Notation:

$$\lim_{a \to 9} \frac{\sqrt{a} - 3}{a - 9} = \lim_{a \to 9} \frac{(\sqrt{a} - 3)(\sqrt{a} + 3)}{(a - 9)(\sqrt{a} + 3)}$$

$$= \lim_{a \to 9} \frac{a - 9}{(a - 9)(\sqrt{a} + 3)}$$

$$= \lim_{a \to 9} \frac{1}{\sqrt{a} + 3}$$

$$= \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

> Do not drop the 'lim' notation until direct substitution is used.

Incorrect Notation:

$$\lim_{a \to 9} \frac{\sqrt{a} - 3}{a - 9} = \frac{(\sqrt{a} - 3)(\sqrt{a} + 3)}{(a - 9)(\sqrt{a} + 3)}$$

$$= \frac{a - 9}{(a - 9)(\sqrt{a} + 3)}$$

$$= \frac{1}{\sqrt{a} + 3}$$

$$= \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

$$\lim_{a \to 9} = \frac{1}{6}$$

> The 'lim' notation has no meaning unless followed by a function expression. Keep the 'lim' notation until direct substitution is used.

Example 2:

Find this limit: $\displaystyle\lim_{x\to\infty} \frac{5x^3 - 7x + 1}{2x^2 - 3x^3}$

Correct Notation:

$$\lim_{x\to\infty} \frac{5x^3 - 7x + 1}{2x^2 - 3x^3} = \frac{5}{-3} = -\frac{5}{3}$$

Incorrect Notation:

$$\lim_{x\to\infty} \frac{5x^3 - 7x + 1}{2x^2 - 3x^3}$$

$$\lim_{x\to\infty} = -\frac{5}{3}$$

Again, the 'lim' notation has no meaning unless followed by an expression. Never write $\displaystyle\lim_{x\to c} =$ a number. Instead write $\displaystyle\lim_{x\to c} f(x) =$ a number.