MA15910 Lesson 27 Notes
(part 1 of section 5.4)

This lesson covers 'curve sketching'. Students often think that this is not important since we have calculators and computers that can immediately show a sketch of a graph. However, a graphing calculator may have limited information in a viewing window. There are other problems also with graphing calculators and computers. And, curve sketching reinforces the learning of the previous lessons and will allow a student to feel more comfortable about the calculus learned so far.

GUIDELINES FOR SKETCHING CURVES

1. Consider the domain of the function, and note any restrictions.

2. Find the *y*-intercept (if it exists) and any *x*-intercepts, if it is not too difficult to do so.

3. If the function if a rational function, find any vertical asymptotes and any horizontal asymptotes. If the function is an exponential function, find any horizontal asymptotes; if it is a logarithmic function, find any vertical asymptotes.

4. Investigate possible symmetry. If $f(-x) = f(x)$ (same equations), the graph is symmetric about the *y*-axis. If $f(-x) = -f(x)$ (opposite signed equations), the function is odd, so the graph is symmetric about the origin.

5. Find the first derivative. Locate any critical values by finding where the first derivative is zero or undefined. Make a first derivative sign chart. Find intervals where the function is increasing or decreasing and any relative maximums or relative minimums.

6. Find the second derivative. Locate possible inflection points by finding where the second derivative is zero or does not exist. Make a second derivative sign chart. Determine intervals where the function is concave upward or concave downward. Find any inflection points.

7. Plot the intercepts, the critical points, the inflection points, the asymptotes, and any other points as needed. Take advantage of any symmetry found in step 4.

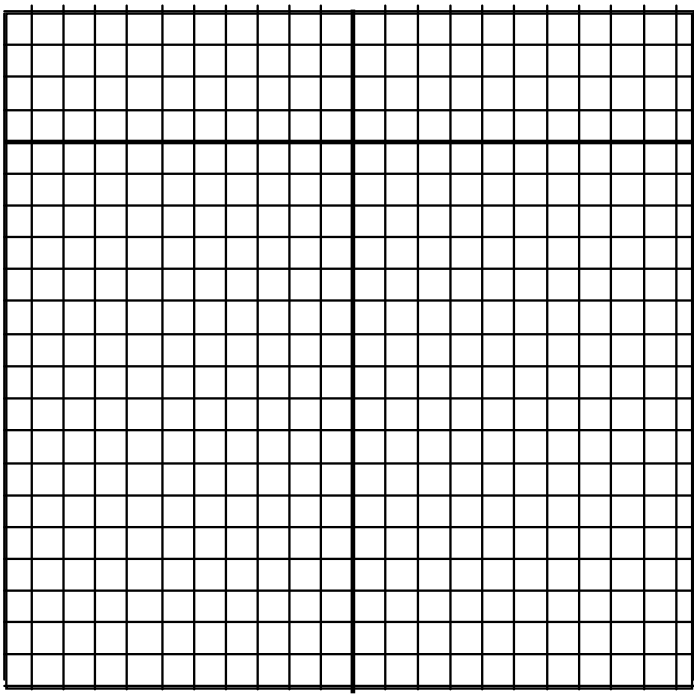
8. Connect the points with a smooth curve using the correct concavity, being careful not to connect points where the function is not defined.

Key hints to sketching reasonable graphs:

1) **Use graph paper or make very, very neat hand-drawn straight axes.**
2) **Use a uniform equally spaced scale on each axis. Choose a scale that is reasonable for each axis.**
3) **Locate any intercepts, relative maximums/minimums or point(s) of inflection.**
4) **Draw any asymptotes.**
5) **Keeping in mind where the graph is increasing/decreasing and concave upward/downward, carefully and as neatly as possible, sketch your graph in pencil.**

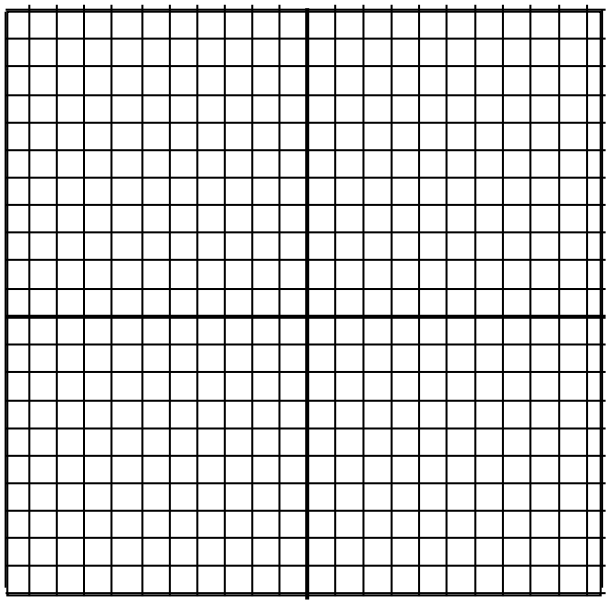
Example 1: Sketch: $y = -x^2 + 2x - 5$

Using the guidelines above, find the relevant information about the function and sketch its graph.



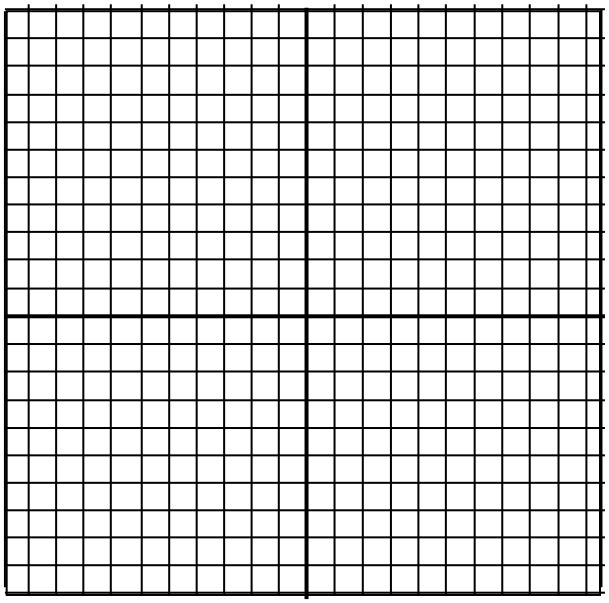
Example 2: $y = 2x^3 + 3x^2 - 12x - 7$

Using the guidelines on the previous page, find the relevant information about the graph of this function and sketch the graph.



Example 3: $g(x) = x^4 + 8x^3 + 18x^2 - 8$

Using the guidelines on a previous page, find the relevant information about the graph of this function and sketch the graph.



Example 4: $y = (x-2)^4$

Using the guidelines on a previous page, find the relevant information about the graph of this function and sketch the graph.