Graphs of Rational Functions II

## Guidelines for Sketching the Graph of a Rational Function:

Assume that $f(x)=\dfrac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials with no common factor.

1. Find the $x$-intercepts – the real zeros of the numerator $g(x)$ – and plot the corresponding points on the $x$-axis.
2. Find the real zeros of the denominator $h(x)$. For each zero $a$, sketch the vertical asymptote $x = a$ with dashes. (Note: If you canceled a common factor, that indicates the location of a 'hole' in the graph.)
3. Find the $y$-intercept $f(0)$, if it exists, and plot the point $(0, f(0))$ on the $y$-axis.
4. Find any horizontal asymptote(s). Sketch in these asymptotes with dashes.
5. If there is a horizontal asymptote $y = c$, determine whether it intersects the graph. The $x$-coordinates of the points of intersection are the solutions of the equation $f(x) = c$. Plot these points, if they exist.
6. Carefully sketch the graph of $f$ in each region determined by the asymptotes. If necessary, use a sign chart to determine where the graph is above or is below the $x$-axis or the horizontal asymptote.

**<u>Summary of finding Asymptotes and Oblique Asymptotes:</u>**

We have discussed how to find vertical asymptotes. Simplify the rational expression representing the rational function. Set the denominator to zero and solve. These are the equations of vertical asymptotes.

We have discussed how to find the location of a 'hole' in the graph. Factor the numerator and denominator of the rational function. If there is a factor that 'cancels', set that factor to zero and that is the location of a 'hole' in the graph.

We have discussed how to find horizontal asymptotes. Compare the term with the highest degree in the numerator to the term with the highest degree in the denominator.

1. If the degree in the numerator is less than the degree in the denominator, the horizontal asymptote is the $x$-axis, or the equation $y = 0$.
2. If the degree in the numerator is equal to the degree in the denominator, the horizontal asymptote is found using the ratio of the coefficients of the leading terms. $y = \frac{a}{b}$, where $a$ is the coefficient of the leading term in the numerator and $b$ is the coefficient of the leading term in the denominator.
3. If the degree in the numerator is greater than the degree in the denominator, there is no horizontal asymptote.

Some rational functions have **<u>slanted or oblique asymptotes.</u>** If a rational function has the form $f(x)=\dfrac{g(x)}{h(x)}$ and if the ***degree of g(x) is one greater than the degree of h(x), then the graph of f(x)*** **has an oblique asymptote. To find the equation of this asymptote, we must use long division of polynomials.** $f(x)=\dfrac{g(x)}{h(x)}=(ax+b)+\dfrac{r(x)}{h(x)}$. **The line $y = ax + b$ from this long division process is the equation of the oblique asymptote.**

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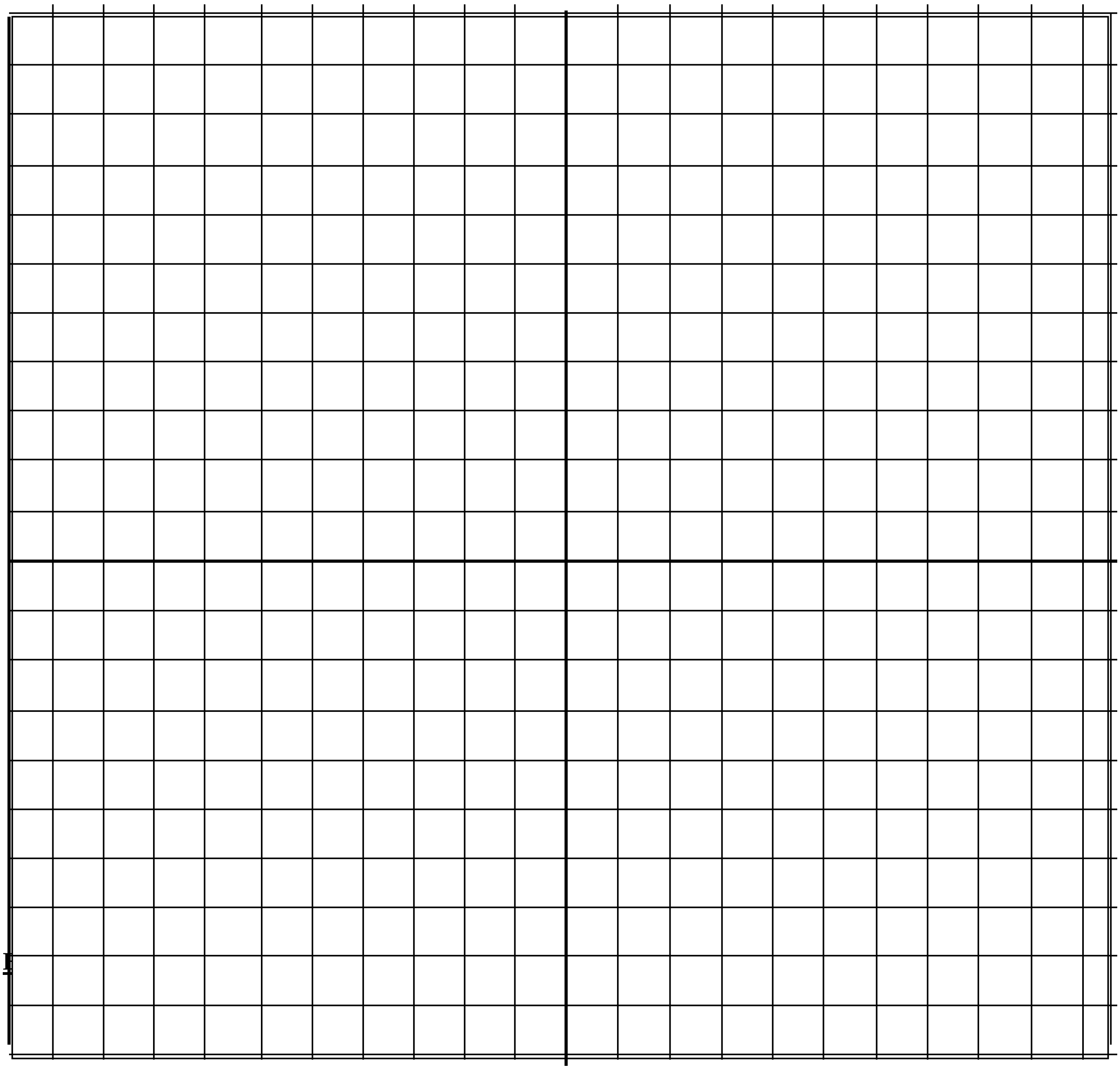
**Ex 0:** Find the equation of any oblique asymptotes.

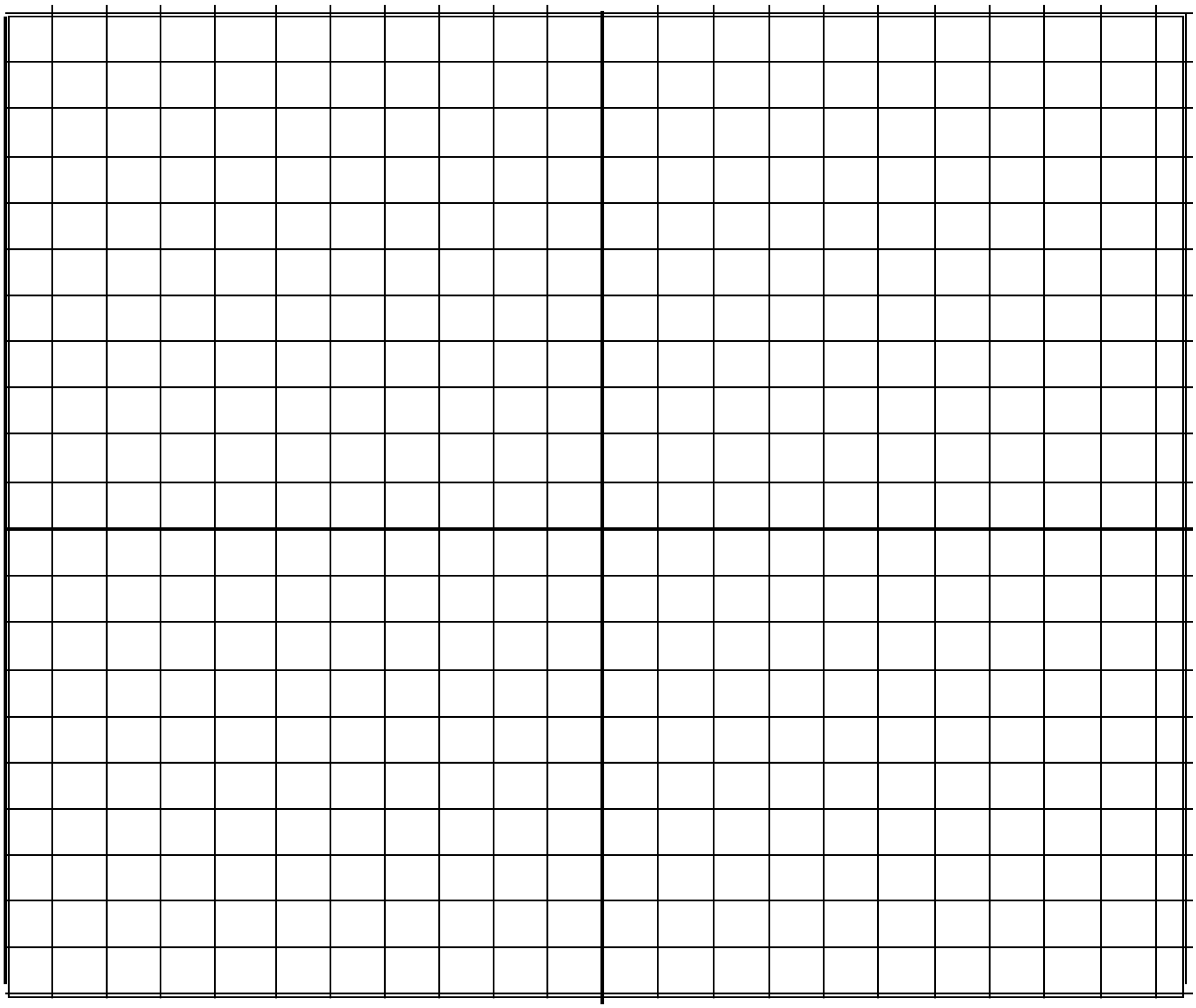
(a) $f(x)=\dfrac{2x^2-x-3}{x-2}$ (b) $g(x)=\dfrac{x^3+1}{x^2-9}$

If we completed all of the examples in lesson 11, here are some additional examples.

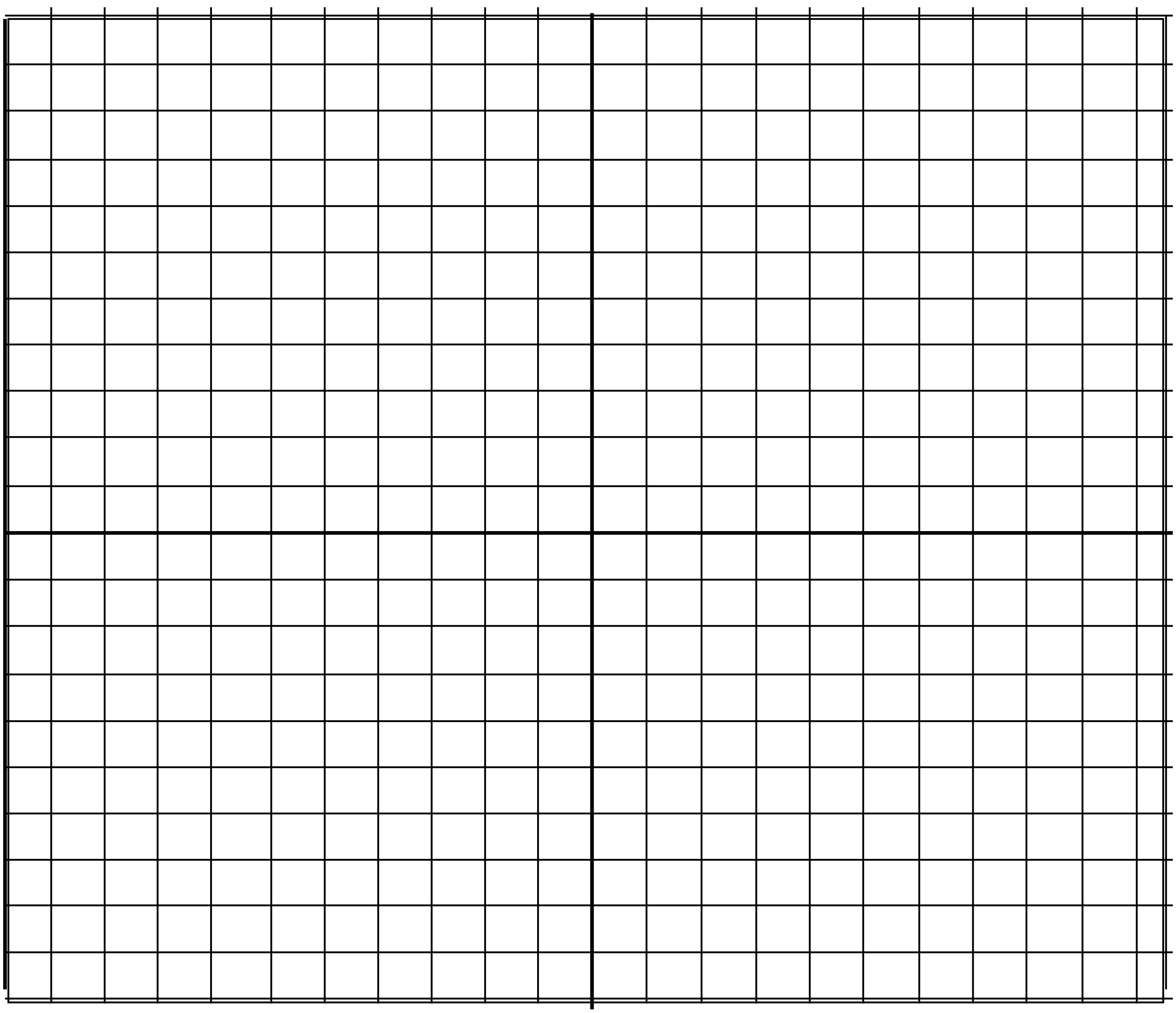
**Ex 1:** $f(x)=\dfrac{4x+3}{2x-5}$ (The graph paper is on the next page.)



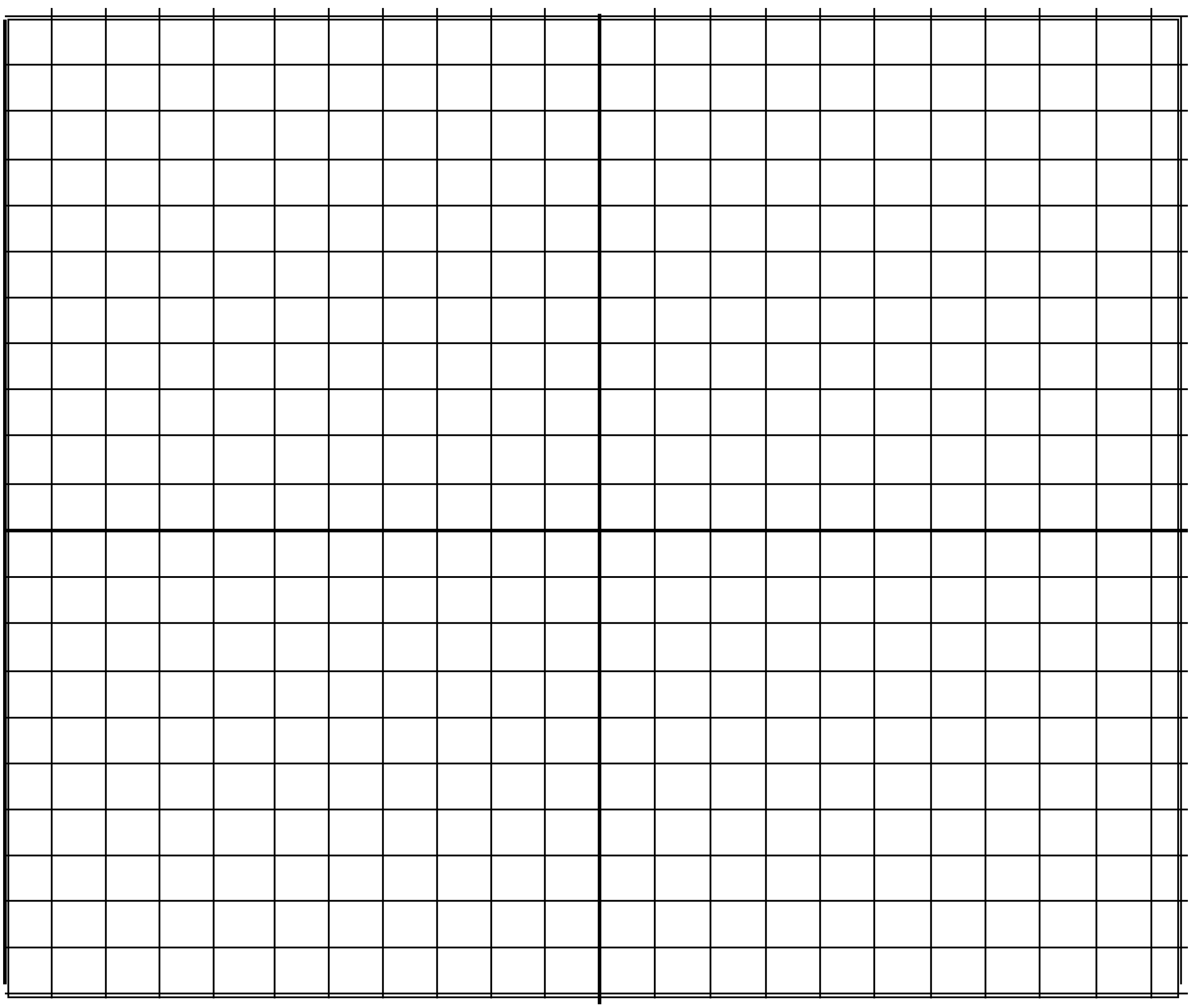
**<u>Ex 2:</u>** $h(x) = \dfrac{3x^2 + x - 4}{2x^2 - 7x + 5}$



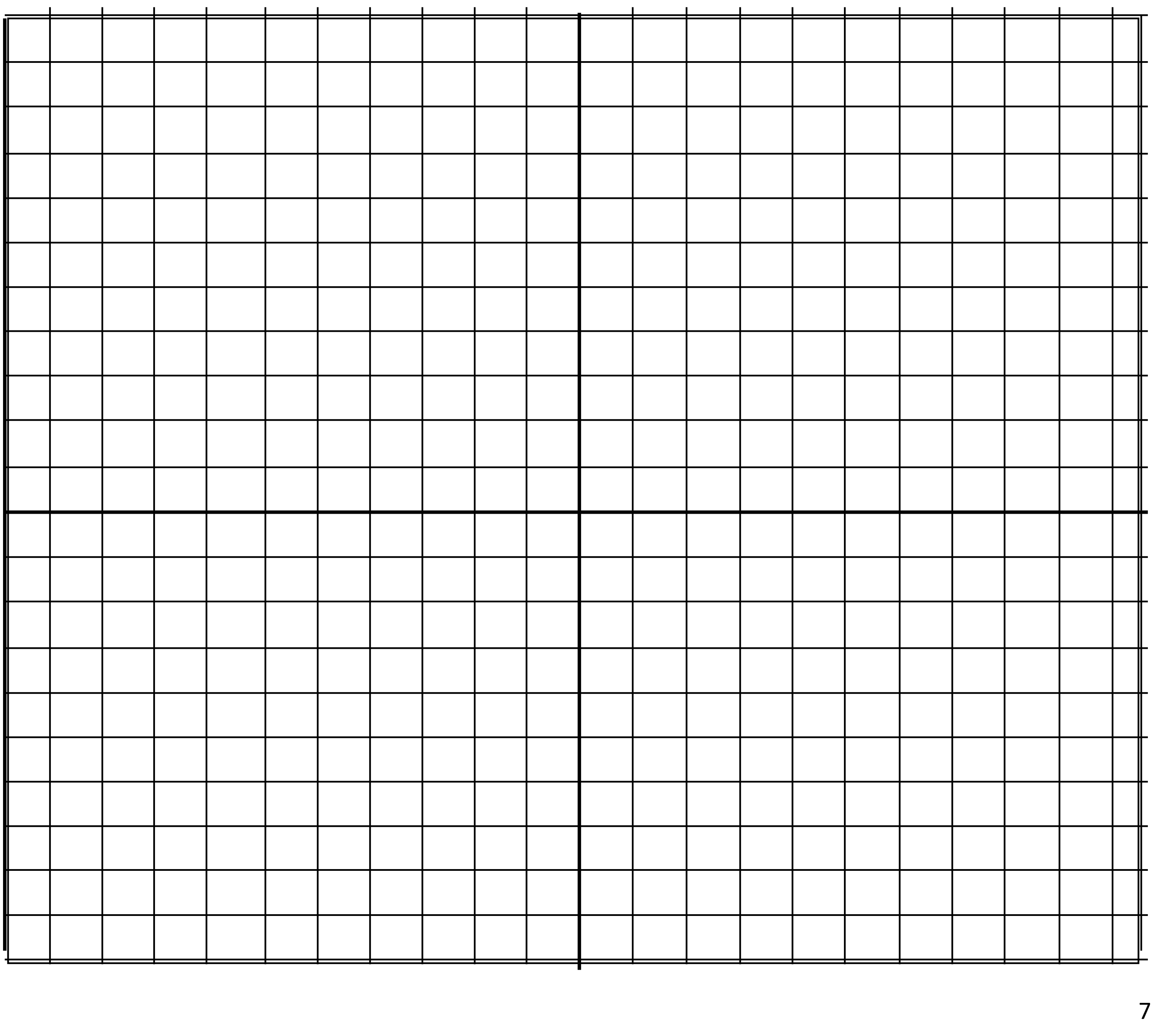
**<u>Ex 3:</u>** $f(x)=\dfrac{x-1}{x^2-x-6}$



**Ex 4:** Graph $y = \dfrac{x^2}{x^2 - x - 2}$



**Ex 5:** Graph: $y = \dfrac{2x^4}{x^4+1}$

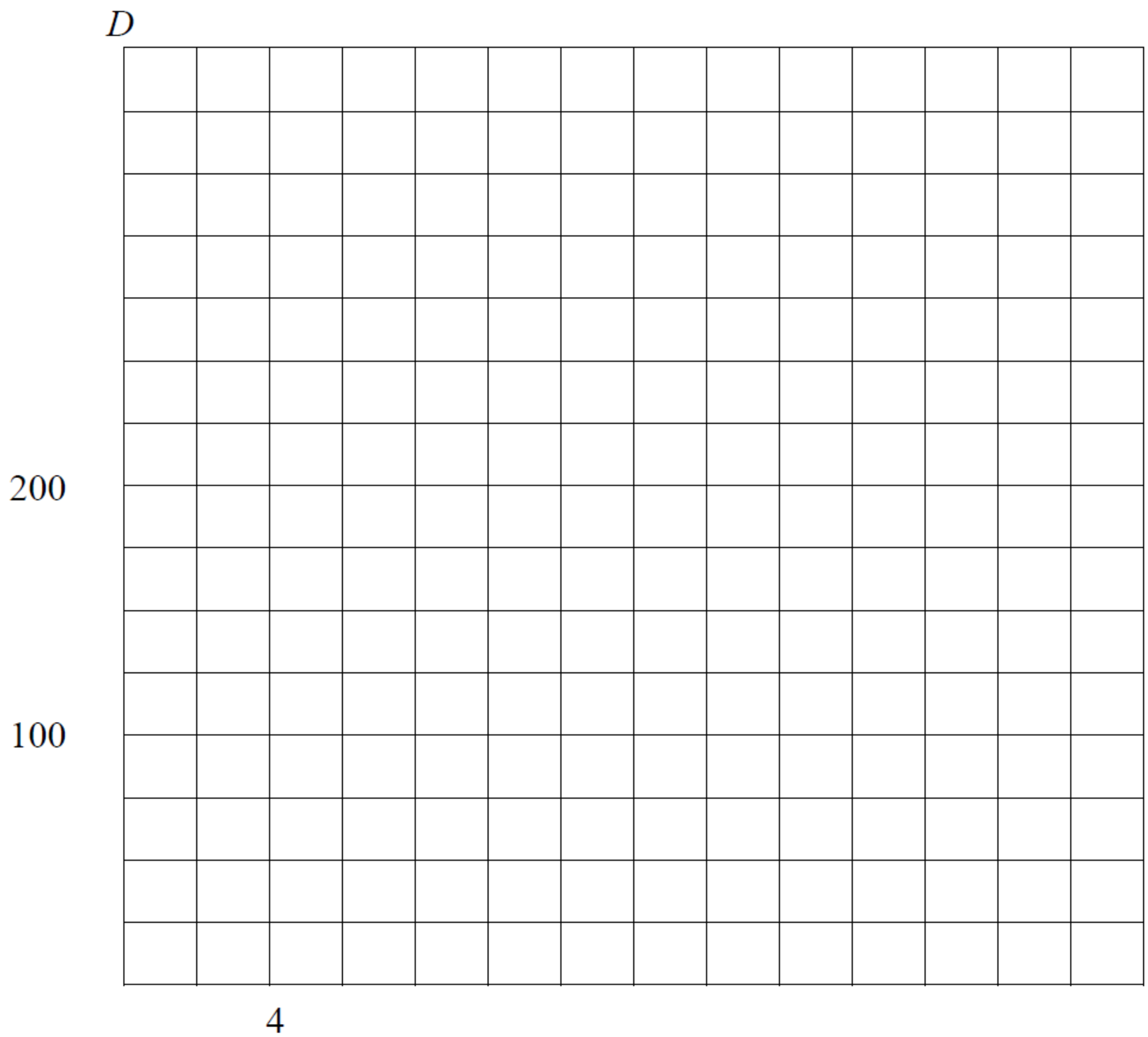


**Ex 6:** Find the equation for a rational function *f* that satisfies these conditions.
Vertical Asymptotes: $x = -4,\ x = 1$
Horizontal Asymptote: $y = -2$
*x*-intercepts (zeros): (3,0), $(-6{,}0)$
Hole at $x = -1$

**Ex 7:** Find the equation for a rational function $g$ that satisfies these conditions.
Vertical Asymptotes: $x = -4,\ x = 2$
Horizontal Asymptote: $y = 0$
*x*-intercept: $(-2{,}0)$
$f(3) = -5$
Hole at $x = 6$

**Ex 8:** The population density $D$ ( in number of people per square mile) in a large city is related to the distance $x$ (in miles) from the center of the city by the function $D(x) = \dfrac{5000x}{x^2 + 36}$.

(a) Sketch a graph of function $D$.



(b) What is happening to the population density as the distance from the center of the city changes from 20 miles to 25 miles?

(c) As the distance from the center of the city continues to increase, what eventually happens to the population density? Think: $x \to \infty$

(d) Use the graph to approximate what areas of the city that have a population density that exceeds 400 people per square mile.