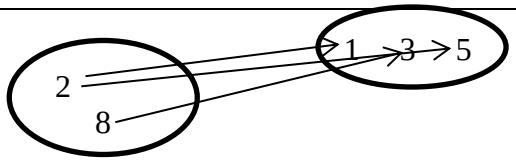
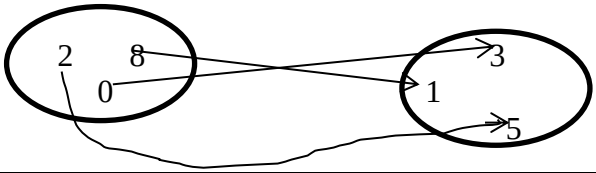
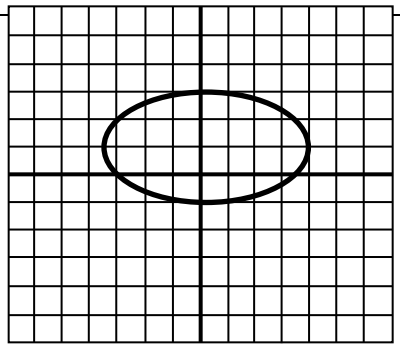
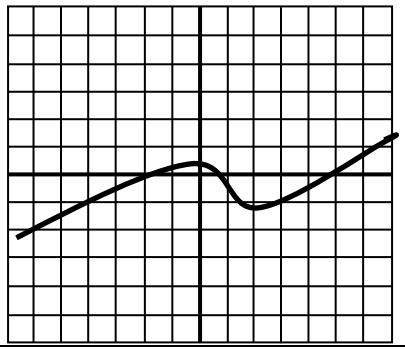
Functions/Domain & Range

Definition of a Function: A function *f* if a correspondence that assigns each element from a **domain set** to one and only one element of a **range set**. Functions may be represented by sets, tables, diagrams (mappings), graphs, and equations. Alternate Definition A function is a set of ordered pairs (x, y) where each x is paired to one and only one y.

Compare the correspondences on the left that are **not functions** with those on the right that **are functions**.

Not Functions	Functions
$\{(2,3), (1,9), (3,0), (1,-5)\}$ (The element 1 from the domain is paired with two different elements from the range.)	$\{(2,3), (1,9), (3,0), (4,-5)\}$ (Each element of the domain is paired with one and only one element of the range.)
x | -2 | -2 | 2 | 4 y | 1 | 2 | 3 | 4 (The element -2 of the domain is paired with both 1 and 2 from the range set.)	x | -2 | 0 | 2 | 4 y | 1 | 2 | 3 | 4 (Each element of the domain is paired with one and only one element of the range.)
	
Think: Vertical Line Test (see below) 	
$y = \pm\sqrt{x},\ x > 0$	$y = \sqrt[3]{x},\quad x \in \mathbb{R}$

DOMAIN: The domain of a function is the set of 'input' values or 'x' values.
RANGE: The range of a function is the set of 'output' values or 'y' values.
In a functions an element of the domain is paired to one and only one element of the range.

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Vertical Line Test: The graph of a set of points in a coordinate plane is the graph of a function if every vertical line intersects the graph in at most one point.

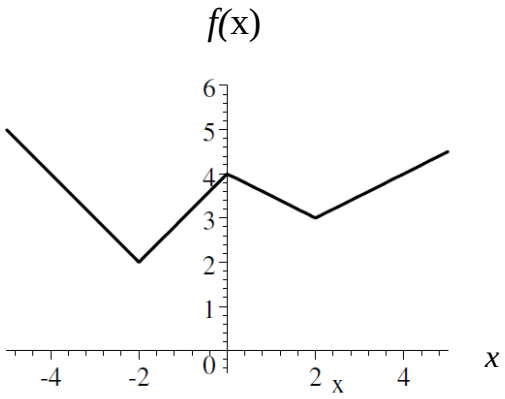
Function Notation: If a function *f* is defined by the rule or correspondence $x - 4x^2$, we can write $y = x - 4x^2$ *or* $f(x) = x - 4x^2$. The notation $f(x)$ is known as 'function notation' and indicates the 'output' of the function *f* that is paired with the 'input' *x*.

Ex 1: Determine if each represents a function of *x*. If it does, write the domain and range.

a)

x	*y*
2	5
7	9
-3	0
8	1
9	1

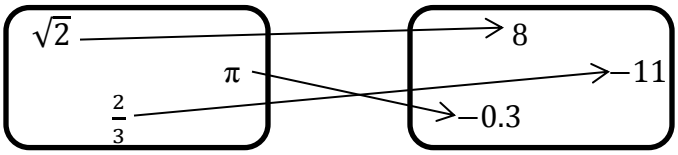
b)



c) $\{(x, y) | y = \sqrt{2x - 1} \ \ x \geq \frac{1}{2}\}$

d) $\{(a, b) | b = \frac{1}{2}a - 1\}$

e)



Ex 2: Given these functions: $h(x) = 2x - \sqrt{x}$, $g(x) = 5x$, $f(x) = 3$; find the following. (Assume the domain of function *h* is $[0, \infty)$ and the domains of function *f* and *g* are 'all real numbers'.) Note: The implied domain of any function is 'all real numbers' or 'only outputs that yield real values' unless given otherwise.

a) $h(4) =$ *b*) $g(\frac{4}{3}) =$ *c*) $f(9) =$

d) $h\left(\frac{16}{9}\right) =$ *e*) $f(\sqrt{2}) =$ *f*) $g(0) =$

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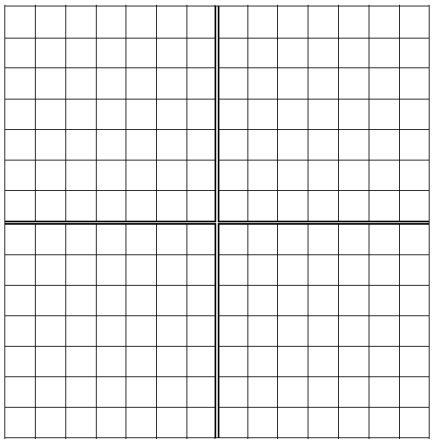
Ex 3: Given: $f(x)=\dfrac{x-4}{x+2}$, $g(x)=x^2-2x+3$, $j(x)=\sqrt{x-4}$. Find the following function values, if they exist.

a) $f(8)=$ *b*) $g(-2)=$ *c*) $j(1)=$

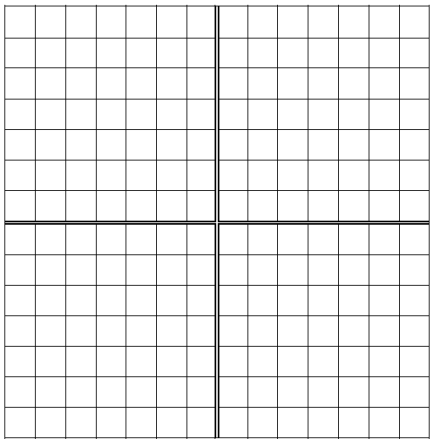
d) $f(-2)=$ *e*) $g(\sqrt{3})=$ *f*) $j(9)=$

Ex 4: Graph each function. Assume the domain only includes 'reasonable' values that yield real 'outputs'. Pick a reasonable scale for each axis.

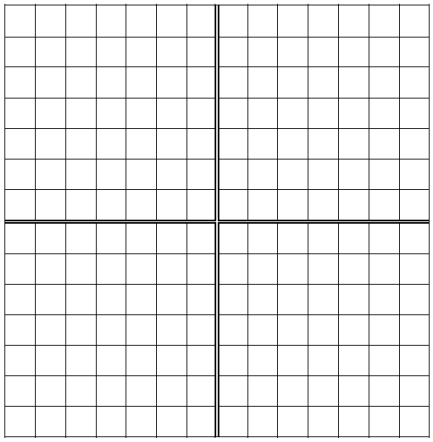
a) $g(x)=\sqrt{2-x}$



b) $f(x)=x^2-3$



c) $y=\frac{3}{2}x-1$



A function *f* is a **linear function** if $\boldsymbol{f(x)=ax+b}$ where *x* is any real number (domain is all real numbers) and *a* and *b* are constants.
The graph of this function at the left will be a **line** with a **slope** of $\frac{3}{2}$ and a **y**-intercept at $(0,-1)$.

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A linear function' graph is a line. An equation for a line can be found if the slope of the line and a point are known or two points are known. Below are some important formulas for line.

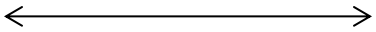
Slope of a Line: If (x_1, y_1) and (x_2, y_2) are two points of a line, then the slope is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

There are four types of slopes.

Positive Slope:
Line rises left to right

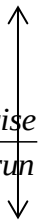
Negative Slope:
Line falls left to right

Zero Slope: (horizontal line)



$$slope = \frac{rise}{run} = \frac{0}{number} = 0$$

Undefined Slope: (vertical line)



$$slope = \frac{rise}{run} = \frac{number}{0}$$ undefined

Equations of Lines:
If a line has slope *m* and a point (x_1, y_1), then
point-slope form of the line is $y - y_1 = m(x - x_1)$.

If a line has slope *m* and a *y*-intercept (0, *b*), then
slope-intercept form of the line is $y = mx + b$.

If *A*, *B*, and *C* represent integers and *A* > 0, then the following are equation of lines.
General Form: $Ax + By + C = 0$
Standard Form: $Ax + By = C$

Ex 5: Find the slope of the line containing points $(-4,2)$ and $\left(\frac{2}{3}, -1\right)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{\frac{2}{3}-(-4)} =$$

Ex 6: If a line has the point-slope form $y-2=\frac{3}{4}(x+5)$, write the equation of the line in general form.

Ex 7: If an equation for a line in standard form is $5x-2y=3$, find the slope and ordered pair for the *y*-intercept of the line.

Ex 8: Find the domain of each function.

a) $f(x)=\sqrt{2x+7}$ *b*) $g(x)=\sqrt{9-x^2}$

c) $h(x)=\frac{2x-3}{x^2-5x+4}$ *d*) $j(x)=\frac{\sqrt{5-x}}{(x-1)(\sqrt{x+2})}$

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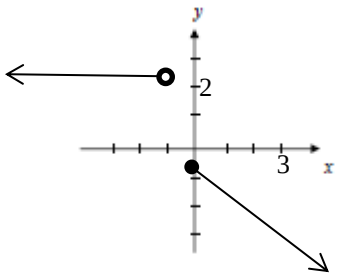
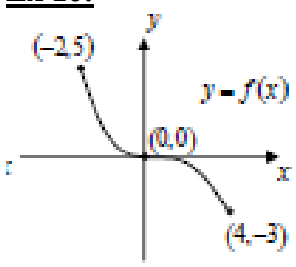
Find the domain and range of each function graphed.

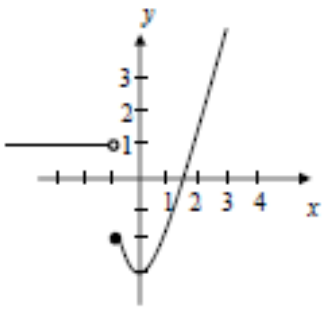
Ex 9:



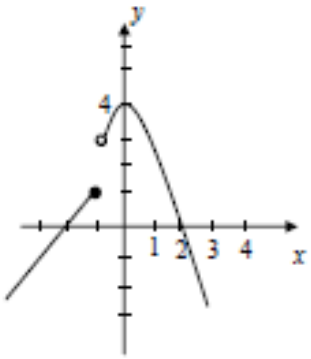
Ex 10:



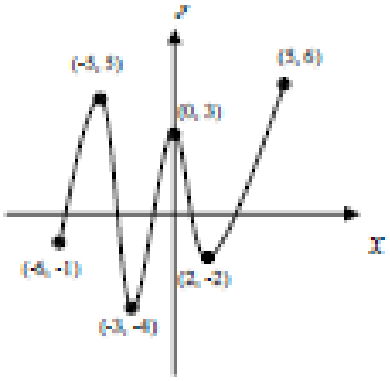
Ex 11:



Ex 12:



Ex 13:



Ex 14: