Study Guide for Exam 1

1. You are supposed to be able to determine the center and radius of a sphere by “completing the square”, given the equation of the form

$$x^2 + y^2 + z^2 + ax + by + cz + d = 0.$$

You are also suppoed to be able to compute the distance between two given points.

Example Poblem: Compute the distance from the point $P = (2, 1, -5)$ to the closest point on the sphere defined by the equation

$$x^2 + y^2 + z^2 + 2x - 6y + 9 = 0,$$

and the distance to the farthest point on the same sphere.

2. You are supposed to be able to compute the dot product $\vec{a} \cdot \vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$. You are supposed to understand the geometrical interpretation of the dot product $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where θ is the angle between the two vectors.

You should be able to use the orthogonality criterion in terms of the dot product

$$\vec{a} \perp \vec{b} \Longleftrightarrow \vec{a} \cdot \vec{b} = 0.$$

3. You are supposed to be able to compute the cross product $\vec{a} \times \vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$. You are supposed to understand the geometrical interpretation of the cross product $\vec{a} \times \vec{b}$ as the vector orthogonal to both $\vec{a}$ and $\vec{b}$, where the direction is determined by the right hand rule, with the magnitude being equal to the area of the parallelogram fromed by the two vectors $\vec{a}$ and $\vec{b}$. (As an application, if you want to compute the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$, then you can just compute the cross product and its magnitude.)

4. You are supposed to be able to compute the vector projection $\mathbf{proj}_{\vec{a}}\vec{b}$ of a vector $\vec{b}$ onto $\vec{a}$, and scalar projection $\mathbf{comp}_{\vec{a}}\vec{b}$ by the formulas

$$\begin{cases} \mathbf{proj}_{\vec{a}}\vec{b} & = & \dfrac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}}\,\vec{a} \\ \mathbf{comp}_{\vec{a}}\vec{b} & = & \dfrac{\vec{a} \cdot \vec{b}}{\sqrt{\vec{a} \cdot \vec{a}}} \end{cases}$$

WARNING: Make a clear distiction between $\mathbf{proj}_{\vec{a}}\vec{b}$ and $\mathbf{proj}_{\vec{b}}\vec{a}$.

5. You are suppoesed to be able to compute the area of the region bounded by two curves $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = b$ by the formula

$$\int_a^b |f(x) - g(x)| dx.$$

6. You are suppoesed to be able to compute the volume of a solid obtained by rotation using **the washer method**.

7. You are suppoesed to be able to compute the volume of a solid obtained by rotation using **the method of cylindrical shells**. (Look at Example 1 in 6.3 on Page 451 of the textbook.)

8. You are suppoesed to be able to compute the volume of a solid, given the description of its base and its cross sections.

9. You are suppoesed to be able to compute the amount of work needed to carry out a task. Typical examples are:

- work needed to empty the water from a tank in the shape of an inverted circulat cone (Look at Example 5 in 6.4 on Page 457 of the textbook.),
- work needed to stretch a spring (Look at Example 3 in 6.4 on Page 457 of the textbook.),
- work needed to lift a chain (Look at Problem 19 in 6.4 on Page 459).

10. You are suppoesed to be able to compute the average value f_{ave} of a function $y = f(x)$ on the interval $[a, b]$ by the formula

$$f_{\text{ave}} = \frac{\int_a^b f(x) dx}{b - a}.$$

11. You are suppoed to be able to evaluate the integral using integration by parts.