

I Solving Linear Equations in One Variable

A **linear equation in one variable** x is an equation that can be simplified or written in the form $ax + b = 0$, where a and b are real numbers, $a \neq 0$. In other words, it is an equation where the variable is only to the first power.

Solving an equation involves determining all values for the variable that result in a true statement when substituted into the equation. We say the value(s) found **satisfy** the equations. The set of all such values that satisfy the equation are listed in the **solution set**.

Equations can be solved by

1. Clearing parentheses using the distributive property.
2. Combining like terms on each side.
3. Adding or Subtracting terms from both sides of the equation so that variable terms are on one side and numbers are on the other side.
4. Multiplying or Dividing both sides by a non-zero number.

***Study Tip:** See the Study Tip on page 103 of the textbook that shows the difference between simplifying an algebraic expression (no equal sign) and solving an algebraic equation. Students must understand the difference between an expression and an equation.

Ex 1: Solve: a) $5x + 3 = 2(3 - 4x)$

$$b) (x - 2)(x - 3) = (x + 3)(x + 4)$$

A linear equation in one variable may contain one or more rational expressions. Since most students do not like working with fractions, finding the solution of the equation is much easier if the denominators are 'cleared' first.

1. Find the LCD.
2. Multiply both sides of the equation by the LCD and distribute to each **term**.
3. Cancel and solve the resulting equation.

Ex 2: Solve the following equations.

$$a) \quad \frac{3+x}{3} + \frac{x+7}{2} = 4x+1$$

$$b) \quad 5 + \frac{x-2}{3} = \frac{x+3}{8}$$

II Solving Rational Equations

If a denominator of an equation includes a variable, it is called a **rational equation**. Clear the denominators of a rational equation as described above. However, as you learned in the previous lesson, a rational expression's domain must exclude any numbers that make factors of the denominator be equal to zero. **You must always determine which numbers are excluded from the domain after you find the LCD. After solving the equation, any result cannot equal those excluded values.** If this happens, there may be **no solution** to the equation. The symbol \emptyset means no solution.

Ex 3: Solve each equation.

$$a) \quad \frac{a}{a+2} - 1 = \frac{3a+2}{a^2+4a+4}$$

$$b) \quad \frac{6}{x+3} - \frac{5}{x-2} = \frac{-20}{x^2+x-6}$$

$$c) \quad \frac{3}{x-2} + \frac{1}{x} = \frac{3}{x-2}$$

III Types of Linear Equations of One Variable

Equations can be placed into categories that depend on their solution sets.

- A An equation that is true for all real numbers for which both sides are defined is called an **identity**. An example is $2x+3+2=x+5+x$. In an identity the right side equals the left side.
- B An equation that is not an identity, but that is true for at least one real number, is called a **conditional equation**. An example is $\frac{4x}{3}+1=x$. This equation is only true if $x=-3$.
- C An equation that is not true for even one real number is an **inconsistent equation**. These are equations that have no solution, such as $x=x+2$. Equations whose only possible solutions lead to zero denominators are included in this category.

Ex 4: Solve, if possible, and categorize each equation as conditional, inconsistent, or identity. Describe the solution as a number, no solution, or all real numbers.

$$a) \quad \frac{1}{x-2} = \frac{2}{x-1} - \frac{1}{3x-3}$$

$$b) \quad 2(a+1) = 3(a-2) - a$$

$$c) \quad 3(x-3) = \frac{6x-18}{2}$$

Ex 5: The equation $C = 865x + 15,316$ models the cost of an item at cost \$10,000 in 1975 for x years after 1980. This is a measure of inflation. Use this model to approximate the cost in 2000. If the actual inflation cost in 2000 was \$32,000, does the model underestimate or overestimate and by how much?