Name: ____________________________ **PUID:** _______________________

Your TA's Name: ________________________ **Recitation Time:** ______________

MA 16100

SPRING 2024

EXAM 3 VERSION A

The problems are numbered 1-10.

For problems 1-8 indicate your answer by filling in the appropriate circle next to the answer choice. Partial credit will **not** be awarded for problems 1-8.

This exam is out of 100 points. Problems 1-8 are worth 8 points each and problems 9 and 10 are worth 16 points each. You will receive 4 points for signing the bottom of this page.

Extra scratch paper is not permitted. Write all your work in this exam booklet.

Write your name and PUID on each page. This will help us locate and successfully grade your test if the pages become separated.

You may not leave the room before 20 minutes have passed. If you finish the exam between when 20 and 50 minutes have passed, you may leave the room after turning in the exam booklet. If you finish within the last 10 minutes of the exam, you MUST REMAIN SEATED until your TA comes and collects your exam booklet.

Exam Policies:

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and the instructor.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or the instructor.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

Student Signature: ____

Name: ______________________________ **PUID:** ____________________

1. (**8 Points**) On which of the following intervals is function $f(x) = x^3 + x^2 - x$ both decreasing and concave up?

   ◯ **A** $\left(-\frac{1}{3}, \frac{1}{3}\right)$.

   ◯ **B** $\left(-1, -\frac{1}{3}\right)$.

   ◯ **C** $\left(\frac{1}{3}, \infty\right)$.

   ◯ **D** $(-\infty, -1)$.

   ◯ **E** $\left(-\frac{1}{3}, \infty\right)$.

Name: ______________________________ **PUID:** ____________________

2. (**8 Points**) Let $f(x)$ be a function continuous on $[2, 7]$ and differentiable on $(2, 7)$ where we have that $f(2) = 1$, $f(4) = 7$, $f(5) = 5$, $f(7) = 1$. Which of the following is **not necessarily** a value of $f'(x)$ for some x in $(2, 7)$?

- ○ **A** 0.
- ○ **B** 6.
- ○ **C** 3.
- ○ **D** $\frac{4}{3}$.
- ○ **E** -2.

Name: ______________________________ **PUID:** ____________________

3. (**8 Points**) How many critical points does the function $f(x) = |x^3 - 3x|$ have?

◯ **A** 1.
◯ **B** 2.
◯ **C** 3.
◯ **D** 4.
◯ **E** 5.

4. (**8 Points**) Let $f(x)$ be differentiable for all real numbers where

$$f'(x) = (x+4)^2(x-1)(x-3)^3(x-5).$$

How many local maxima does $f(x)$ have?

- ◯ **A** 0.
- ◯ **B** 1.
- ◯ **C** 2.
- ◯ **D** 3.
- ◯ **E** 4.

Name: ______________________________ **PUID:** ____________________

5. (**8 Points**) How many local minima does the function $f(x) = x^4 - \dfrac{4}{3}x^3 - 12x^2$ have?

   ◯ **A** 0.

   ◯ **B** 1.

   ◯ **C** 2.

   ◯ **D** 3.

   ◯ **E** 4.

6. (**8 Points**) A farmer needs to plant a rectangular field directly along a river. To protect the field, they need to build a fence around three of its sides (the fourth side along the river does not require a fence). Suppose the farmer has materials to build 1200m of fencing. If ℓ denotes the length of the side of the fence parallel to the river and w denotes the length of the sides of the fence perpendicular to the river, what dimensions of the field will maximize its area?

   ○ **A** $\ell = 200, w = 500.$

   ○ **B** $\ell = 400, w = 400.$

   ○ **C** $\ell = 600, w = 300.$

   ○ **D** $\ell = 800, w = 200.$

   ○ **E** $\ell = 1000, w = 100.$

Name: ______________________________ **PUID:** ____________________

7. (**8 Points**) Which of the following is the best linear approximation to $y = \sqrt{x}$ at the point $x = 16$.

   ◯ **A** $y = \frac{1}{2}x + 2$

   ◯ **B** $y = \frac{1}{4}x$

   ◯ **C** $y = \frac{1}{8}x + 4$

   ◯ **D** $y = \frac{1}{2}x - 4$

   ◯ **E** $y = \frac{1}{8}x + 2$

8. (**8 Points**) Compute $\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^x$.

   ◯ **A** e^3.

   ◯ **B** ∞.

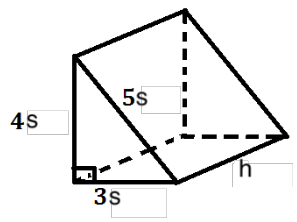
   ◯ **C** 0.

   ◯ **D** e^{-3}.

   ◯ **E** 1.

Name: ______________________________ **PUID:** ____________________

9. A company wants create a closed cardboard box which is shaped like a triangular prism with a right triangle whose sides are in ratio 3 to 4 to 5 as its base and top. Let h denote the height of the box and let s be such that the length of the sides of the triangular base and top of the box are $3s$, $4s$ and $5s$, respectively. The diagram below gives a bottom view of the box.



(i) (**3 Points**)
Write down a formula for the volume of the box in terms of h and s.

$V =$ ____________________________________

(ii) (**3 Points**)
Write down a formula for the surface area of the box in terms of h and s.

$A =$ ____________________________________

(iii) (**4 Points**) The company wishes to build the box using 12 square metres of cardboard. Given this restriction, write a formula for the volume of the box in terms of s. Show all of your work.

$V(s) =$ ____________________________________

Name: ______________________________ **PUID:** ____________________

(*Note: this is a continuation of Problem 9.*)

(iv) (**6 Points**) If the box is to be built with 12 square metres of cardboard, find the value of s which maximizes the box's internal volume. Show all of your work.

$s =$________________________

Name: ______________________________ **PUID:** ____________________

10. Let $f(x) = \dfrac{1}{x+1} + \dfrac{1}{x-1}$.

    (i) (**2 Points**) Determine if $f(x)$ has any vertical asymptotes (write NONE if it has none). Show your work.

    The vertical asymptotes are ____________________________________

    (ii) (**2 Points**) Determine if $f(x)$ has any horizontal asymptotes (write NONE if it has none). Show your work.

    The horizontal asymptotes are ____________________________________

Name: ____________________________ **PUID:** ____________________

(*Note: this is a continuation of Problem 10.*)

(iii) (**1 Point**) Find all points x at which $f(x) = 0$.

$f(x) = 0$ for $x =$ ______________________

(write "NONE" if $f(x) \neq 0$ for any x)

(iv) (**3 Points**) Determine when $f(x)$ is increasing and decreasing.

$f(x)$ is increasing on __________________________________

(write "N/A" if the function does not increase on any interval)

$f(x)$ is decreasing on __________________________________

(write "N/A" if the function does not decrease on any interval)

Name: ____________________________ **PUID:** __________________

(*Note: this is a continuation of Problem 10.*)

(v) (**3 Points**) Determine when $f(x)$ is concave up and concave down.

$f(x)$ is concave up on __________________________________

(write "N/A" if the function is not concave up on any interval)

$f(x)$ is concave down on __________________________________

(write "N/A" if the function is not concave down on any interval)

(vi) (**1 Point**) Find all inflection points of $f(x)$.

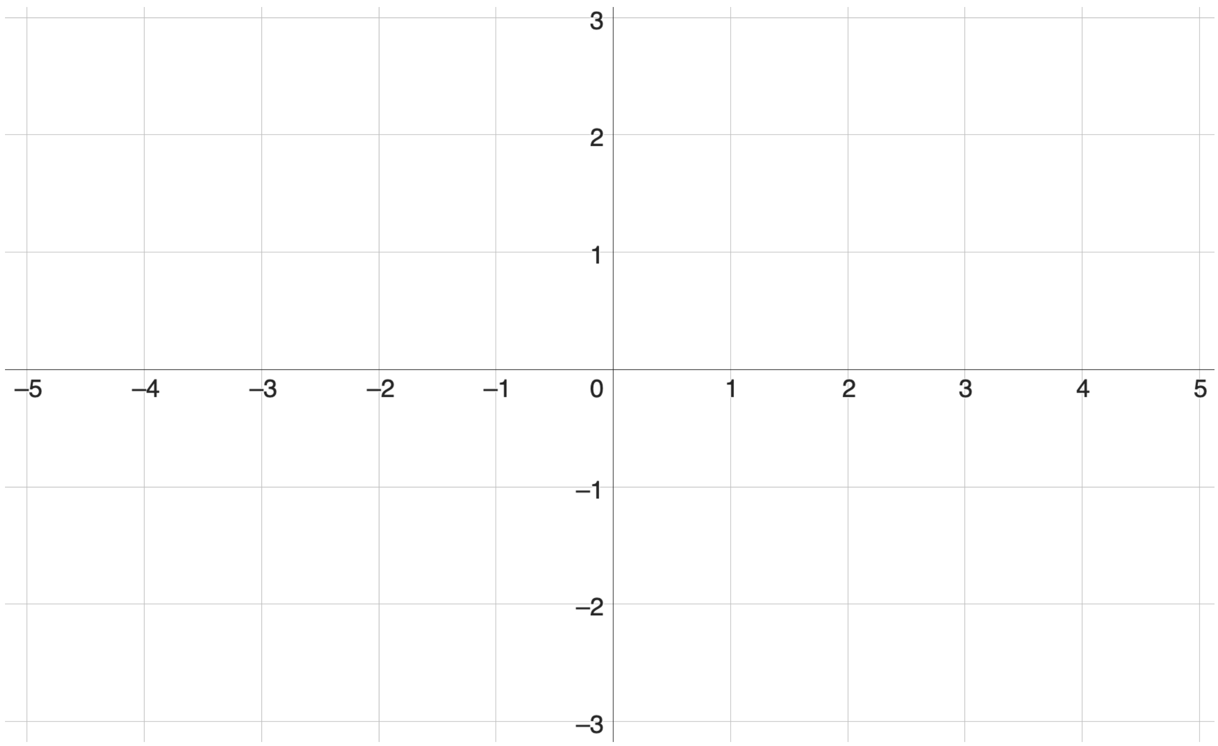
$f(x)$ has an inflection point at ______________________

(write "NONE" if $f(x)$ has no inflection points)

Name: ______________________________ **PUID:** ____________________

(*Note: this is a continuation of Problem 10.*)

(vii) (**4 Points**) Draw a rough graph of $f(x)$ on the following set of axes indicating the information you determined in parts (i)-(vi).



(*This page may be used for scratch work but work done here will not be graded.*)