MA 16100 - Final Exam - 12/11/2023
TEST NUMBER: 11 - (GREEN Booklet)

NAME _____ **YOUR TA'S NAME** _____

STUDENT ID # _____ **RECITATION TIME** _____

Be sure the paper you are looking at right now is GREEN and matched with the color of the scantron! Write **11** in the TEST/QUIZ NUMBER boxes and blacken the appropriate spaces on the scantron. Use a #2 pencil for the scantron and fill in:

1. Your name. If there's not enough space, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: **11**
4. Student Identification Number: **Your Purdue ID Number with two leading zeros**

There are **25** questions, each worth 4 points. Blacken your answer choice on the scantron for questions 1-25. Use this exam booklet for all your work and use the back of the test pages for scrap paper. Submit both the scantron and the exam booklet when finished.

If you finish the exam before 9:50 AM, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20 AM. If you don't finish by 9:50 AM, you MUST REMAIN SEATED until your TA collects your materials.

EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. At what point(s) (x, y) on the graph of $f(x) = \dfrac{x}{x+5}$ does the tangent line to $f(x)$ have a slope of $\frac{1}{5}$?

A. $(0, 0)$ and $(5, \frac{1}{2})$

B. There are no such points

C. $(0, 0)$ and $(-10, 2)$

D. $(10, \frac{1}{3})$ only

E. $(0, 0)$ only

2. Find $\displaystyle\int \frac{\sin x \cos x}{1 + \sin^2 x}\, dx$

A. $\dfrac{\tan^{-1}(\sin x)}{2} + C$

B. $\ln(|1 + \sin x|) + C$

C. $\ln\left(\sqrt{1 + \sin^2 x}\right) + C$

D. $\tan^{-1}(\sin x) + C$

E. $\ln(1 + \sin^2 x) + C$

3. If $g(x) = \displaystyle\int_0^{x^3} \sin(2t)\ dt$, then $g'(x) =$

A. $2x^3 \sin(2x^3)$

B. $\sin(2x)$

C. $\sin(2x^3)$

D. $6x^2 \sin(2x)$

E. $3x^2 \sin(2x^3)$

4. Which of the following statements are true about the function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x$?

I. $f(x)$ is increasing on $(-2, 1)$
II. $f(x)$ is concave up on $(-\frac{1}{2}, \infty)$
III. $f(x)$ has one inflection point

A. (II) and (III) only

B. (I) only

C. (I), (II), and (III)

D. (II) only

E. (I) and (II) only

5. A 5 foot ladder standing on level ground leans against a vertical wall. The bottom of the ladder is pulled away from the wall at 2 ft/sec. How fast is the AREA under the ladder changing when the top of the ladder is 4 feet above the ground?

A. -6 ft^2/sec

B. $\frac{7}{4}$ ft^2/sec

C. $-\frac{25}{4}$ ft^2/sec

D. $-\frac{3}{2}$ ft^2/sec

E. 25 ft^2/sec

6. Find the equation of the tangent line to the curve $2y^4 - x^2y = x^3$ at the point $(1, 1)$.

A. $y = \frac{5}{8}x + \frac{3}{8}$

B. $y = \frac{5}{7}x + \frac{2}{7}$

C. $y = \frac{1}{3}x + \frac{2}{3}$

D. $y = \frac{5}{9}x + \frac{4}{9}$

E. $y = x$

7. Let $f(x) = 5x^2 - 7x$. Use the definition of the derivative to find $f'(1)$. When you simplify the terms inside the limit, you get:

A. $f'(1) = \lim_{h\to 0}(3 + 7h)$

B. $f'(1) = \lim_{h\to 0}(3 + 3h)$

C. $f'(1) = \lim_{h\to 0}(3 + h)$

D. $f'(1) = \lim_{h\to 0}(3 + 5h)$

E. $f'(1) = \lim_{h\to 0}(3 - 2h)$

8. Compute the limit: $\lim_{x\to 0} \dfrac{3e^x - 1}{\sin x}$

A. 0

B. 3

C. ∞

D. Limit does not exist

E. $-\infty$

9. Consider a function $f(x)$ defined such that $\int_3^{11} f(x)\,dx = 15,$ and $\int_7^{11} f(x)\,dx = 11.$

What is $\int_7^3 3f(x)dx$?

A. -4

B. -15

C. 12

D. 8

E. -12

10. Which of the following functions has $\lim_{x\to\infty} f(x) = 2$?

A. $f(x) = \dfrac{1+x^2}{x}$

B. $f(x) = \dfrac{1+x}{2x}$

C. $f(x) = \dfrac{1}{2x-1}$

D. $f(x) = \dfrac{1+2x}{x}$

E. $f(x) = \dfrac{1-2x}{x}$

11. Suppose that $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = 0$. What can be said about $\lim_{x\to 0} f(x)g(x)$?

A. $\lim_{x\to 0} f(x)g(x) = 0$

B. The limit may or may not exist. If it exists, it can be any number

C. $\lim_{x\to 0} f(x)g(x) = \infty$

D. $\lim_{x\to 0} f(x)g(x)$ is either 0 or ∞

E. $\lim_{x\to 0} f(x)g(x) = 1$

12. A rectangle with sides parallel to the axes is inscribed in the region above the x-axis and below the parabola $y = 12 - x^2$. The maximum area of such a rectangle is

A. 32

B. 6

C. 24

D. 48

E. 12

13. Find the limit. $\lim_{x \to 2^+} \left(\ln \frac{x}{2}\right) \left(\tan \frac{\pi x}{4}\right)$

A. $-\frac{\pi}{4}$

B. ∞

C. 0

D. $-\frac{4}{\pi}$

E. $-\infty$

14. Find the integral

$$\int_{e^{25}}^{e^{49}} \frac{1}{x\sqrt{\ln(x)}}\, dx$$

A. 4

B. 1

C. 2

D. 6

E. 3

15. Suppose $f(x) = \dfrac{x+3}{9-x^2}$. Then

A. $\lim\limits_{x\to 3^-} f(x) = \infty$; $\lim\limits_{x\to -3} f(x)$ does not exist.

B. $\lim\limits_{x\to 3^-} f(x) = \dfrac{1}{6}$; $\lim\limits_{x\to -3} f(x) = -\infty$.

C. $\lim\limits_{x\to 3^-} f(x) = -\infty$; $\lim\limits_{x\to -3} f(x) = \dfrac{1}{6}$.

D. $\lim\limits_{x\to 3^-} f(x) = \infty$; $\lim\limits_{x\to -3} f(x) = \dfrac{1}{6}$.

E. $\lim\limits_{x\to 3^-} f(x) = -\infty$; $\lim\limits_{x\to -3} f(x)$ does not exist.

16. Express the given quantity as a single logarithm

$$\frac{3}{2}\ln(x+4) - \ln(\sqrt{x}) - \frac{1}{2}\ln(x^3+10)$$

.

A. $\dfrac{\ln\left(\frac{x(x+4)^3}{(x^3+10)}\right)}{2}$

B. $\ln\left(\sqrt{\dfrac{(x+4)^3}{\sqrt{x}(x^3+10)}}\right)$

C. $\ln\left(\sqrt{\dfrac{(x+4)}{x(x^3+10)}}\right)$

D. $\ln\left(\dfrac{(x+4)^3}{x(x^3+10)}\right)$

E. $\ln\left(\sqrt{\dfrac{(x+4)^3}{x(x^3+10)}}\right)$

17. A certain population of bacteria is growing exponentially. At time $t = 0$, there are 100 bacteria and at time $t = 1$ the population doubles to 200 bacteria. At what time will there be 300 bacteria present?

A. $\dfrac{\ln 2}{3}$

B. $\ln\left(\dfrac{2}{3}\right)$

C. $\dfrac{\ln 2}{\ln 3}$

D. $\ln\left(\dfrac{3}{2}\right)$

E. $\dfrac{\ln 3}{\ln 2}$

18. Suppose $f(x) = 5 + \sqrt[3]{2x - 8}$, find $f^{-1}(x)$.

A. $\dfrac{1}{5 + \sqrt[3]{2x - 8}}$

B. $\dfrac{(x - 5)^3 + 8}{2}$

C. $(x - 5)^3 + 8$

D. $\dfrac{\sqrt[3]{x - 5} + 8}{2}$

E. $\dfrac{(x + 8)^3 - 5}{2}$

19. A farmer wants to fence off a rectangular field that borders a river on one side. No fence is required along the river. If the area of the field is to be 100 square feet, what is the least amount of fence (in feet) required?

A. 30

B. 100

C. $2\sqrt{50}$

D. $5\sqrt{50}$

E. $4\sqrt{50}$

20. Suppose $f(x) = \sqrt{3}\sin x + \cos x$. If M is the absolute maximum of f on the interval $[0, \pi]$, and m is the absolute minimum value on the same interval, what is the sum $M + m$?

A. 2

B. 3

C. 0

D. $\sqrt{3} - 1$

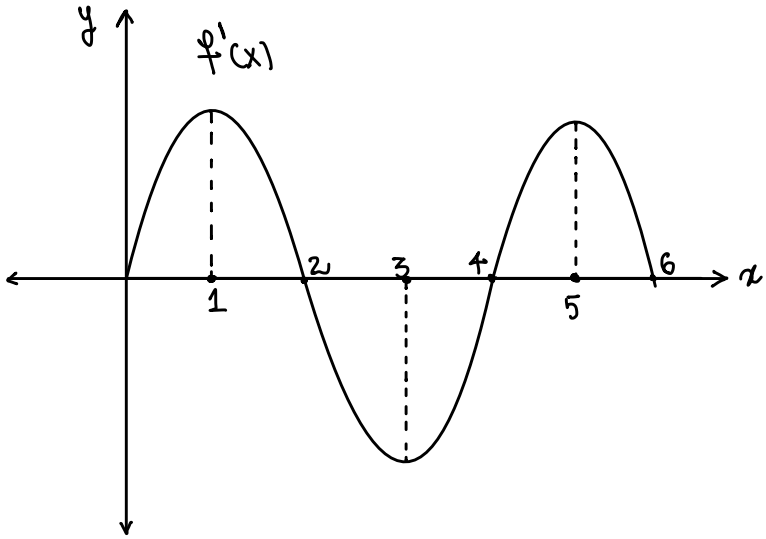
E. 1

21. If f is a differentiable function on $(-\infty, \infty)$, $f(2) = 4$, and $f(6) = 8$, which of the following statements must be true?

I. There is a c in $(2, 6)$ such that $f(c) = 6$.
II. There is a c in $(2, 6)$ such that $f'(c) = 6$.
III. There is a c in $(2, 6)$ such that $f'(c) = 1$.

A. Only I and II
B. Only I and III
C. Only II and III
D. Only II
E. Only I

22. Consider a function $f(x)$. The graph of its **derivative** is plotted below.



Which of the following statements is true?

I. $f(x)$ is decreasing on $(2, 4)$
II. $f(x)$ has a local minimum at $x = 3$
III. $f(x)$ is concave down on $(0, 2)$

A. I and II only
B. I, II and III
C. I only
D. II and III only
E. I and III only

23. A spherical balloon is inflated at a constant rate of 100 in^3/s . How fast, in in/s, is the radius of the balloon increasing when the radius is 5 in? Recall the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

A. $\frac{1}{2\pi}$
B. $\frac{3}{5\pi}$
C. $\frac{1}{\pi}$
D. $\frac{2}{\pi}$
E. $\frac{3}{\pi}$

24. Suppose $g(e) = 4$ and $g'(e) = 2$. If $y = x^{g(x)}$, then what is y' at $x = e$?

A. $4e^3$
B. $8e^3$
C. $\dfrac{4}{e}$
D. $2e^4 + 4e^3$
E. $\frac{4}{e} + 2e^4$

25. How many vertical and horizontal asymptotes does the following function have?

$$f(x) = \frac{x^3 + 21}{x^3 - 4x^2}$$

A. 2 horizontal and 3 vertical

B. 1 horizontal and 2 vertical

C. 0 horizontal and 2 vertical

D. 1 horizontal and 1 vertical

E. 0 horizontal and 3 vertical

(This page left intentionally blank for scratch work.)

(This page left intentionally blank for scratch work.)

(This page left intentionally blank for scratch work.)

(This page left intentionally blank for scratch work.)