# MA 16200: First Midterm Examination
## Spring 2024, Purdue University

Exam version: 01

Name: _____  PUID #: _____

**Instruction:**

- Follow these instructions carefully. Failure to do so may results in your exam being invalidated and/or an academic integrity violation. All suspected violation of academic integrity will be reported to the Office of the Dean of Students.

- Mark your recitation section below. Write your name and PUID on the top of this cover page. **DO NOT WRITE ANYTHING ELSE** on this cover sheet.

| ✓ | Sec # | Time | TA Name | | ✓ | Sec # | Time | TA Name |
|---|-------|------|---------|---|---|-------|------|---------|
| | 0027 | 7:30AM | Nathan Kapsin | | | 0016 | 12:30PM | Tanmay Devale |
| | 0050 | 7:30AM | Brian Wen | | | 0018 | 12:30PM | Risa Fines |
| | 0028 | 8:30AM | Nathan Kapsin | | | 0023 | 12:30PM | Cian Nolan |
| | 0029 | 8:30AM | Brian Wen | | | 0015 | 1:30PM | Tanmay Devale |
| | 0048 | 8:30AM | Sina Nadi | | | 0017 | 1:30PM | Risa Fines |
| | 0052 | 8:30AM | Ali Sheikh | | | 0024 | 1:30PM | Cian Nolan |
| | 0046 | 8:30AM | Aaron Thomas | | | 0031 | 1:30PM | Mary Collins |
| | 0049 | 9:30AM | Sina Nadi | | | 0030 | 2:30PM | Mary Collins |
| | 0053 | 9:30AM | Ali Sheikh | | | 0014 | 2:30PM | Madison Sullivan |
| | 0047 | 9:30AM | Aaron Thomas | | | 0013 | 3:30PM | Madison Sullivan |
| | 0051 | 10:30AM | Mohit Pandiya | | | 0025 | 3:30PM | Conner Partaker |
| | 0032 | 11:30AM | Mohit Pandiya | | | 0026 | 4:30PM | Conner Partaker |

- Use a **#2 PENCIL** to mark the scantron sheet. Fill in the following information:
  - **Your Name:** If there are not enough spaces, fill in as much as you can.
  - **Section Number:** Use all four digits as indicated in the table above.
  - **Test Number:** Fill in 01 for this version of exam.
  - **Student Identification Number:** Fill in your 10-digit PUID with two leading zeros.
  - Write down your TA's name and sign the scantron sheet.
  - Black in your answers in the spaces provided for questions 1–12.

- Do not open the exam booklet or start writing before the proctor signals the start of the exam.

- Do all your work in this exam booklet. Use the back sides of the exam booklet for scratch work.

- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.

- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.

- Turn in both the scantron sheet and the exam booklet when you are finished.

- If you finish the exam before 7:25 pm, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50 pm. If you don't finish before 7:25 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your scantron sheet and your exam booklet. You must stop working when the proctor signals the end of exam.

*This exam consists of 12 questions. Each question is worth the same number of points. You have exactly one hour to finish the exam. Good luck!*

## Questions:

1. Consider the sphere $(x-1)^2 + (y-2)^2 + (z+4)^2 = 9$. Which of the following is true?

   (A) The sphere intersects the $xy$-plane but not the $yz$-plane or the $xz$-plane.

   (B) The sphere intersects the $xy$-plane and the $yz$-plane but does not intersect the $xz$-plane.

   (C) The sphere intersects the $xz$-plane but not the $yz$-plane or the $xy$-plane.

   (D) The sphere intersects the $xz$-plane and the $yz$-plane but does not intersect the $xy$-plane.

   (E) The sphere intersects the $xz$-plane and the $xy$-plane but does not intersect the $yz$-plane.

2. A cubic tank whose sides are 1 m long sits on the ground and is filled with a liquid of density 100 kg/m$^3$. Assume gravitational acceleration $g = 10$ m/s$^2$, compute the the total force on one of its side walls due to pressure.

   (A) 300 N

   (B) 500 N

   (C) 800 N

   (D) 200 N

   (E) 400 N

3. Find the area of the region enclosed by $x = y^2$ and $x = 8 - y^2$.

(A) $\dfrac{8}{3}$

(B) $\dfrac{16}{3}$

(C) $\dfrac{64}{3}$

(D) $\dfrac{4}{3}$

(E) $\dfrac{32}{3}$

4. An object weighing 30 N is suspended by a spring from the ceiling. The spring is stretched beyond its equilibrium position by 0.1 m due to the weight of the object. How much work is needed to return the spring to its equilibrium position?

(A) 1.5 J

(B) 15 J

(C) 30 J

(D) 3 J

(E) 10 J

5. What is the angle between the vectors $\vec{a} = 3\vec{i}$ and $\vec{b} = -\vec{i} - \sqrt{2}\vec{j} + \vec{k}$.

   (A) $2\pi/3$

   (B) $\pi/3$

   (C) $0$

   (D) $5\pi/6$

   (E) $\pi/6$

6. What is the area of the triangle with vertices $(1, 1, 1)$, $(3, -1, 2)$, and $(5, 1, -2)$?

   (A) $10\sqrt{2}$

   (B) $3$
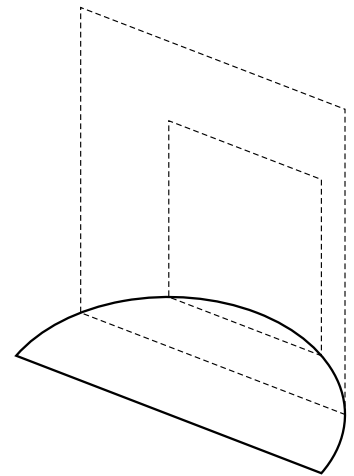
   (C) $10$

   (D) $5\sqrt{2}$

   (E) $5$

7. Let $R$ be the region bounded by $y = x^2$, $y = 2$, and the $y$-axis. What is the volume of the solid obtained by revolving $R$ around the $y$-axis?

(A) $\dfrac{8\pi}{3}$

(B) $2\pi$

(C) $\dfrac{2\sqrt{2}\pi}{3}$

(D) $\pi$

(E) $\dfrac{4\sqrt{2}\pi}{3}$

8. A solid has a semicircular base of radius 3. Cross sections perpendicular to the base and parallel to the straight edge of the semicircle are squares. What is the volume of the solid?

(A) $\dfrac{9\pi}{2}$

(B) $72$

(C) $18$

(D) $27\pi$

(E) $\dfrac{9\pi}{4}$

9. Find the surface area of the surface generated when the curve

$$y = \frac{x^3}{3}, \quad 0 \le x \le 1$$

is rotated about the $x$-axis.

(A) $\dfrac{\pi(2\sqrt{2}-1)}{27}$

(B) $\dfrac{\pi(2\sqrt{2}-1)}{9}$

(C) $\dfrac{\pi(2\sqrt{2}-1)}{3}$

(D) $\dfrac{\pi(2\sqrt{2}-1)}{287}$

(E) $\dfrac{\pi(2\sqrt{2}-1)}{81}$

10. Which one of the following integrals equals the arc length of the curve

$$y = x^3 + 1, \quad 1 \le x \le 2 \quad ?$$

(A) $\displaystyle\int_1^2 x^3 + 1\, dx$

(B) $\displaystyle\int_1^2 \sqrt{9x^4 + 1}\, dx$

(C) $\displaystyle\int_1^2 3x^2\, dx$

(D) $\displaystyle\int_1^2 \sqrt{x^3 + 2}\, dx$

(E) $\displaystyle\int_1^2 \sqrt{x^3 + 1}\, dx$

11. Which one of the following integrals equals the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the line $x = 3$?

(A) $\displaystyle\int_0^1 2\pi(3 - x)(1 - x^2)dx$

(B) $\displaystyle\int_0^1 2\pi(3 - x)^2(x - x^2)^2 dx$

(C) $\displaystyle\int_0^1 2\pi(3 - x)(x - x^2)^2 dx$

(D) $\displaystyle\int_0^1 2\pi(3 - x)(x - x^2)dx$

(E) $\displaystyle\int_0^1 2\pi(3 - x^2)(x - x^2)dx$

12. A thin metal bar of length 2 m pivots around a hinge on the floor. At $x$ meters away from the hinge, the bar has density $\rho(x) = 10 + x$ kg/m. Assume gravitational acceleration $g = 10$ m/s². How much work *against gravity* is required to pivot the bar upright from a horizontal position?

(A) 22 joules

(B) 220 joules

(C) 440 joules

(D) $\dfrac{520}{3}$ joules

(E) $\dfrac{680}{3}$ joules
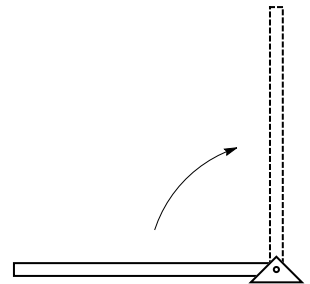
**DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.**
This page is left blank intentionally for scratch work.