

MA 16200
EXAM 3 Form 01
November 9, 2023

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below the boxes on the scantron: 01

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: 01
4. Student Identification Number: **This is your Purdue ID number with two leading zeros.**

There are **12** questions, each worth 8 points (you will automatically earn 4 points for filling out your student ID number correctly). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 7:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50. If you don't finish before 7:20, you **MUST REMAIN SEATED** until your TA comes to collect your scantron and your exam.

EXAM POLICIES

1. Please do NOT open the exam until instructed to do so.
2. Please obey the instructions and requests by all proctors, TAs, and lecturers.
3. To minimize distractions, you may NOT leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, please put down all writing instruments and remain in your seat. Your TA will collect your scantron and the exam. Once you have turned in your scantron and exam, you are free to leave.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. Which of the following sequences are convergent?

I. $\left\{ a_k = \frac{2k}{3k+5} \right\}$

II. $\{ a_k = \cos k\pi \}$

III. $\left\{ a_k = k \tan \left(\frac{1}{k} \right) \right\}$

- A. I and II only
B. II and III only
C. I and III only
D. I only
E. All of them
2. Determine if the sequence $\left\{ a_k = \sqrt[k]{k^2} + \frac{(-1)^k \sqrt{k}}{k+1} \right\}$ for $k = 1, 2, 3, \dots$ converges. And if it does, find what it converges to.

- A. converges to 1
B. converges to 2
C. converges to 0
D. converges to -1
E. the sequence diverges

3. Find the sum of the following convergent telescoping series

$$\sum_{k=1}^{\infty} \frac{3}{k(k+3)} = \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+3} \right)$$

- A. $\frac{5}{2}$
B. $\frac{7}{6}$
C. $\frac{7}{12}$
D. $\frac{11}{6}$
E. $\frac{7}{4}$
4. The series

$$\sum_{k=1}^{\infty} \frac{1}{(k^{3p} + 1)^{1/6}}$$

is convergent if and only if

- A. $p > \frac{1}{6}$
B. $p > \frac{1}{3}$
C. $p > \frac{1}{2}$
D. $p > 2$
E. $p > 3$

5. Find the sum of the following convergent series

$$\sum_{k=0}^{\infty} (-1)^k \frac{8}{3^k}$$

- A. 12
B. 6
C. 4
D. 3
E. 2
6. Which of the following series converge by **the integral test**?

I. $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$

II. $\sum_{k=2}^{\infty} \frac{1}{k \ln(k^2)}$

III. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

- A. II and III only
B. III only
C. II only
D. I only
E. none of them

7. Which of the following series converge by the **limit comparison test**?

I. $\sum_{k=2}^{\infty} \frac{1}{\sqrt{k}-1}$

II. $\sum_{k=1}^{\infty} \frac{2^k + 1}{3^k}$

III. $\sum_{k=4}^{\infty} \frac{k+15}{k(k-3)(k+2)}$

- A. All of them
 - B. None of them
 - C. III only
 - D. I and II only
 - E. II and III only
8. Consider the series

$$\sum_{k=1}^{\infty} \frac{10 + 7 \cos(k\pi)}{k^2 + \sqrt{k}}$$

Which of the following statements is correct?

A. The series converges by a comparison to $\sum_{k=1}^{\infty} \frac{17}{k^2}$

B. The series diverges by a comparison to $\sum_{k=1}^{\infty} \frac{17}{\sqrt{k}}$

C. The series diverges by a comparison to $\sum_{k=1}^{\infty} \frac{3}{k^2}$

D. The series converges by a comparison to $\sum_{k=1}^{\infty} \frac{17}{\sqrt{k}}$

E. The series diverges because $\lim_{k \rightarrow \infty} \frac{10 + 7 \cos(k\pi)}{k^2 + \sqrt{k}}$ does not exist

9. What is the minimum number of terms needed to estimate the sum of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^3}$ with $|\text{error}| < \frac{1}{1000}$ (that is, the absolute value of the error is **strictly less than** $\frac{1}{1000}$)?

- A. 9
- B. 10
- C. 11
- D. 1000
- E. 999

10. Consider the following two alternating series:

I. $\sum_{k=1}^{\infty} (-1)^k \frac{\pi}{k^4}$

II. $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{k}{k^2 + 4k + 5}$

Which of the following statements is true?

- A. I and II are absolutely convergent
- B. I is conditionally convergent and II is absolutely convergent
- C. I and II are conditionally convergent
- D. I is absolutely convergent and II is divergent
- E. I is absolutely convergent and II is conditionally convergent

11. For **how many** of the following series does the **root test** show that the series converges?

I. $\sum_{k=1}^{\infty} \left(\frac{k}{k+5} \right)^{2k^2}$

II. $\sum_{k=1}^{\infty} \left(1 + \frac{19}{k} \right)^{k^2}$

III. $\sum_{k=1}^{\infty} \left(\frac{k-4}{k} \right)^k$

IV. $\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k^2} \right)^k$

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

12. On which of the following intervals does the series $\sum_{k=1}^{\infty} \frac{3x^k}{2k^2}$ converge?

- A. $-1 \leq x \leq 1$
- B. $-1 < x \leq 1$
- C. $-1 < x < 1$
- D. $-1 \leq x < 1$
- E. Converges for all x