MA 16200: Third Midterm Examination
Fall 2024, Purdue University

Exam version: 01

Name: ______________________________ PUID #: ____________________

Exam Instructions:

- Follow these instructions carefully. Failure to do so may result in your exam being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.
- Mark the circle of your recitation section below. Write your name and PUID on the top of this cover page. **DO NOT WRITE ANYTHING ELSE** on this cover page.

	Sec	Time	TA Name		Sec	Time	TA Name
◯	206	7:30AM	Gage Bachmann	◯	214	10:30AM	Claudia Phagan
◯	109	7:30AM	Lance Daley	◯	113	11:30AM	Tausif Ahmed
◯	904	7:30AM	Luca Mossman	◯	105	11:30AM	Otto Baier
◯	906	7:30AM	Michael Poole	◯	115	12:30PM	Tausif Ahmed
◯	210	7:30AM	Ehan Shah	◯	101	12:30PM	Alexis Cruz Castillo
◯	208	8:30AM	Gage Bachmann	◯	103	1:30PM	Alexis Cruz Castillo
◯	111	8:30AM	Lance Daley	◯	218	1:30PM	Leo Shen
◯	908	8:30AM	Luca Mossman	◯	220	2:30PM	Leo Shen
◯	902	8:30AM	Michael Poole	◯	117	3:30PM	Tifany Burnett
◯	212	8:30AM	Ehan Shah	◯	204	3:30PM	Mohamad Mousa
◯	224	9:30AM	Niveditha Nerella	◯	121	3:30PM	Juliet Raginsky
◯	216	9:30AM	Claudia Phagan	◯	119	4:30PM	Tifany Burnett
◯	107	10:30AM	Otto Baier	◯	202	4:30PM	Mohamad Mousa
◯	222	10:30AM	Niveditha Nerella	◯	123	4:30PM	Juliet Raginsky

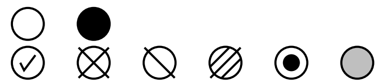
- This exam consists of 11 questions for a total of 100 points.
- You have exactly one hour to complete the exam.
- Do not open the exam booklet or start writing before the proctor signals the start of the exam.
- Write your PUID on every other page of the exam booklet. This will help us locate your test if the pages become separated. Only do this after the exam starts.
- Additional pages for scratch work can be found at the end of the booklet.
- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.
- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.
- **DO NOT DETACH ANY PAGES** from the exam booklet.
- If you finish the exam before 8:55 pm, you may leave the room after turning in the exam booklet. You may not leave the room before 8:20 pm. If you don't finish before 8:55 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your exam booklet. You must stop working when the proctor signals the end of exam.

Good luck!

Multiple-choice Instructions:

- For multiple choice questions, fill the circles completely with a **#2 PENCIL** for your answer choices. If you need to change your answer choice, erase the mark completely.

DO:
DON'T:



- Partial credit will not be awarded for multiple choice questions.

Fill-in-the-blank Instructions:

- For fill-in-the-blank questions, write your answers in the provided text boxes. Answers written entirely or partially outside of the boxes will not be graded.
- Only write the final answer in text boxes. Intermediate steps and scratch work should be completed in the blank space below each question.
- Write all your answers in one line. Fractions of the form $\dfrac{X}{Y}$ are okay to include.

DO: $\boxed{10\pi}$ $\boxed{\frac{2x}{3} + \sin(x)}$

DON'T: $\boxed{5 \cdot 2\pi}$ $\boxed{\phantom{\frac{2x}{3}}}$ $\frac{2x}{3} + \sin(x)$ $\boxed{\begin{array}{l}\frac{2x}{3} - (-\sin(x)) \\ \quad = \frac{2x}{3} + \sin(x)\end{array}}$ $\boxed{\begin{array}{l}\frac{2x}{3} \\ + \sin(x)\end{array}}$

- Partial credit will not be awarded for individual answer boxes. You may get partial credit for fill-in-the-blank questions if there are multiple text boxes in one question.

Common Maclaurin series:

$$
\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots &&= \sum_{k=0}^{\infty} \frac{x^k}{k!}, &&\text{for } -\infty < x < \infty \\
\sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, &&\text{for } -\infty < x < \infty \\
\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, &&\text{for } -\infty < x < \infty \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots &&= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, &&\text{for } -1 < x \le 1 \\
\arctan(x) &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, &&\text{for } -1 \le x \le 1
\end{aligned}
$$

1. (8 points) What is the result of using the third-order Taylor polynomial for $f(x) = e^x$ centered at the origin to approximate the value of e^{-2}?

- ○ (A) $1/3$
- ○ (B) $19/3$
- ○ (C) 0
- ○ (D) 1
- ○ (E) $-1/3$

2. (8 points) Which of the following conclusion is correct if the **ratio test** is applied to the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \quad ?$$

- ○ (A) The series is conditionally convergent because $r = 1$.
- ○ (B) The series is absolutely convergent because $r = 0$.
- ○ (C) The series is divergent because $r = 4$.
- ○ (D) The series is absolutely convergent because $r = 1/4$.
- ○ (E) The ratio test is inconclusive because $r = 1$.

3. (8 points) Which one of the following functions has the Maclaurin series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k} \quad ?$$

- ◯ (A) $2\cos(x)$
- ◯ (B) e^{-2x}
- ◯ (C) $e^{-x/2}$
- ◯ (D) e^{-x^2}
- ◯ (E) $\cos(2x)$

4. (8 points) What is the radius of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{k+1}{k!}(x+1)^k \quad ?$$

- ◯ (A) ∞
- ◯ (B) 1
- ◯ (C) 2
- ◯ (D) 1/2
- ◯ (E) 0

5. (8 points) Which one of the following is a power series for the function $f(x) = \dfrac{2}{2+x}$?

- (A) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k} x^k$
- (B) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{2^k} x^k$
- (C) $\displaystyle\sum_{k=0}^{\infty} (-1)^k 2^k x^k$
- (D) $\displaystyle\sum_{k=0}^{\infty} (-1)^k x^{2k}$
- (E) $\displaystyle\sum_{k=0}^{\infty} 2^k x^k$

6. (8 points) What is the third-order Taylor polynomial for $f(x) = \sin(x)$ centered at $a = \pi/3$?

- (A) $x - \dfrac{1}{6}x^3$
- (B) $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}x - \dfrac{\sqrt{3}}{2}x^2 - \dfrac{1}{2}x^3$
- (C) $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}x - \dfrac{\sqrt{3}}{4}x^2 - \dfrac{1}{12}x^3$
- (D) $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\left(x - \dfrac{\pi}{3}\right) - \dfrac{\sqrt{3}}{2}\left(x - \dfrac{\pi}{3}\right)^2 - \dfrac{1}{2}\left(x - \dfrac{\pi}{3}\right)^3$
- (E) $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\left(x - \dfrac{\pi}{3}\right) - \dfrac{\sqrt{3}}{4}\left(x - \dfrac{\pi}{3}\right)^2 - \dfrac{1}{12}\left(x - \dfrac{\pi}{3}\right)^3$

7. (8 points) How many of the following convergence tests are applied correctly to detemine that the following series is divergent?

$$\sum_{n=2}^{\infty} \frac{2n}{n^2-1}$$

- The series is divergent by the **limit comparison test** with $\sum \frac{1}{n}$.
- The series is divergent by the **ratio test**.
- The series is divergent by the **integral test**.
- The series is divergent by the **divergence test**.

○ (A) 0
○ (B) 1
○ (C) 2
○ (D) 3
○ (E) 4

8. (8 points) Evaluate the sum of the series

$$S = \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n-1}.$$

Hint: $S = f(1/2)$ if $f(x) = \sum_{n=1}^{\infty} nx^{n-1}$.

○ (A) 8
○ (B) 2
○ (C) 4
○ (D) 1
○ (E) The series is divergent.

9. (8 points) Which of the following conclusion is correct if the **root test** is applied to the series

$$\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{-n^2} \quad ?$$

- ◯ (A) The series is divergent because $\rho = e$.
- ◯ (B) The root test is inconclusive because $\rho = 1$.
- ◯ (C) The series is absolutely convergent because $\rho = 0$.
- ◯ (D) The series is divergent because $\rho = \infty$.
- ◯ (E) The series is absolutely convergent because $\rho = 1/e$.

10. (8 points) If we use the second-order Taylor polynomial $p_2(x)$ for $f(x) = \ln(1+x)$ centered at the origin to approximate the value of $\ln(1.1)$, which one of the following statements is true about the remainder $R_2(0.1)$?

The derivatives of $f(x)$ are listed below:

$$f'(x) = \frac{1}{1+x} \qquad f''(x) = -\frac{1}{(1+x)^2} \qquad f'''(x) = \frac{2}{(1+x)^3}$$

- ◯ (A) $|R_2(0.1)| = \dfrac{|f''(c)|}{2!} \cdot |0.1|^2 \le \dfrac{1}{2!} \cdot (0.1)^2$ for some $0 \le c \le 0.1$.
- ◯ (B) $|R_2(0.1)| = \dfrac{|f'''(c)|}{3!} \cdot |0.1|^3 \le \dfrac{2}{3!} \cdot (0.1)^3$ for some $0 \le c \le 0.1$.
- ◯ (C) $|R_2(0.1)| = \dfrac{|f''(c)|}{2!} \cdot |0.1|^2 \le \dfrac{1/(1.1)^2}{2!} \cdot (0.1)^2$ for some $0 \le c \le 0.1$.
- ◯ (D) $|R_2(0.1)| = \dfrac{|f'''(c)|}{3!} \cdot |0.1|^3 \le \dfrac{2/(1.1)^3}{3!} \cdot (0.1)^3$ for some $0 \le c \le 0.1$.
- ◯ (E) None of the above.

11. This question is about the function $f(x)$ represented by its Maclaurin series

$$f(x) = \sum_{k=0}^{\infty} \frac{k+1}{2^k} x^k.$$

Note: For all parts of this question, if the evaluation is infinite, write either $+\infty$ or $-\infty$ accordingly. If the evaluation does not exist and does not approach one of the two infinities, write "DNE". If the evaluation is $+\infty$ or $-\infty$, writing "DNE" will not receive credit.

(a) (6 points) The radius of convergence for the Maclaurin series of $f(x)$ is $R = \boxed{}$.

(b) (4 points) The interval of convergence for the Maclaurin series of $f(x)$ is

- ○ (A) $(-R, R)$
- ○ (B) $[-R, R]$
- ○ (C) $[-R, R)$
- ○ (D) $(-R, R]$

Note: "R" in the choices above represents the radius of convergence from part (a).

(c) (5 points) The third derivative $f^{(3)}(0) = \boxed{}$.

(d) (5 points) The definite integral $\int_0^1 f(x)\,dx = \boxed{}$.

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