MA 16200: Third Midterm Examination

Spring 2024, Purdue University

Exam version: 01

Name: ______________________ PUID #: ______________________

Instruction:

- Follow these instructions carefully. Failure to do so may results in your exam being invalidated and/or an academic integrity violation. All suspected violation of academic integrity will be reported to the Office of the Dean of Students.
- Mark your recitation section below. Write your name and PUID on the top of this cover page. **DO NOT WRITE ANYTHING ELSE** on this cover sheet.

✓	Sec #	Time	TA Name
	0027	7:30AM	Nathan Kapsin
	0050	7:30AM	Brian Wen
	0028	8:30AM	Nathan Kapsin
	0029	8:30AM	Brian Wen
	0048	8:30AM	Sina Nadi
	0052	8:30AM	Ali Sheikh
	0046	8:30AM	Aaron Thomas
	0049	9:30AM	Sina Nadi
	0053	9:30AM	Ali Sheikh
	0047	9:30AM	Aaron Thomas
	0051	10:30AM	Mohit Pandiya
	0032	11:30AM	Mohit Pandiya

✓	Sec #	Time	TA Name
	0016	12:30PM	Tanmay Devale
	0018	12:30PM	Risa Fines
	0023	12:30PM	Cian Nolan
	0015	1:30PM	Tanmay Devale
	0017	1:30PM	Risa Fines
	0024	1:30PM	Cian Nolan
	0031	1:30PM	Mary Collins
	0030	2:30PM	Mary Collins
	0014	2:30PM	Madison Sullivan
	0013	3:30PM	Madison Sullivan
	0025	3:30PM	Conner Partaker
	0026	4:30PM	Conner Partaker

- Use a **#2 PENCIL** to mark the scantron sheet. Fill in the following information:
  - **Your Name:** If there are not enough spaces, fill in as much as you can.
  - **Section Number:** Use all four digits as indicated in the table above.
  - **Test Number:** Fill in `01` for this version of exam.
  - **Student Identification Number:** Fill in your 10-digit PUID with two leading zeros.
  - Write down your TA's name and sign the scantron sheet.
  - Black in your answers in the spaces provided for questions 1–12.
- Do not open the exam booklet or start writing before the proctor signals the start of the exam.
- Do all your work in this exam booklet. Use the back sides of the exam booklet for scratch work.
- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.
- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.
- Turn in both the scantron sheet and the exam booklet when you are finished.
- If you finish the exam before 8:55 pm, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 6:50 pm. If you don't finish before 8:55 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your scantron sheet and your exam booklet. You must stop working when the proctor signals the end of exam.

This exam consists of 12 questions. Each question is worth 1 point. You have exactly one hour to finish the exam. Good luck!

Questions:

1. Three statements are given.

   (I) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both convergent sequences, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a convergent sequence.

   (II) If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are both divergent sequences, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a divergent sequence.

   (III) If $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence and $\{b_n\}_{n=1}^{\infty}$ is a divergent sequence, then $\{a_n + b_n\}_{n=1}^{\infty}$ is a divergent sequence.

   Select all TRUE statement(s) from above.

   (A) Both (I) and (III)
   (B) Both (I) and (II)
   (C) Both (II) and (III)
   (D) All (I), (II), and (III)
   (E) Only (I)

2. Three statements are given.

   (I) If $\lim\limits_{n\to\infty} a_n$ does not exist, then $\sum\limits_{n=1}^{\infty} a_n$ is divergent.

   (II) If $\sum\limits_{n=1}^{\infty} a_n$ is divergent, then $\lim\limits_{n\to\infty} a_n \neq 0$.

   (III) If $\lim\limits_{n\to\infty} a_n = \dfrac{1}{2}$, then $\sum\limits_{n=1}^{\infty} a_n$ is divergent.

   Select all TRUE statement(s) from above.

   (A) All (I), (II), and (III)
   (B) Only (I)
   (C) Only (II)
   (D) Only (III)
   (E) Both (I) and (III)

3. The sequence $\{a_n\}_{n=1}^{\infty}$ is defined by the recurrence relation

$$a_1 = 1, \quad \text{and} \quad a_{n+1} = (a_n)^2 - 1.$$

What is a_5?

(A) 3
(B) 31
(C) -1
(D) -3
(E) 1

4. The series

$$\sum_{k=1}^{\infty} \frac{(-1)^k \arctan(k)}{k!}$$

is

(A) divergent by the test for divergence.
(B) divergent by the integral test.
(C) divergent by the ratio test.
(D) absolutely convergent by the ratio test.
(E) conditionally convergent.

5. What are all values of p that will make the following series converge?

$$\sum_{n=1}^{\infty} \frac{(\ln(n))^p}{n}$$

(A) $p > -1$
(B) $p < -1$
(C) p can be any real number.
(D) There is no value of p that makes the series converge.
(E) $p \geq -1$

6. Find the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}.$$

(A) 1/3
(B) 2/7
(C) 2/5
(D) 1/5
(E) 1/7

7. Three statements are given about a series $\sum_{n=1}^{\infty} a_n$ with all $a_n \geq 0$.

(I) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} (a_n)^n$ is divergent.

(II) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} (a_n)^n$ is convergent.

(III) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} (a_n)^2$ is convergent.

Select all TRUE statement(s) from above.

(A) All (I), (II), and (III)

(B) Only (II)

(C) Only (I)

(D) Both (II) and (III)

(E) Both (I) and (II)

8. Find the sum of the series

$$\sum_{m=4}^{\infty} \left(\frac{2}{3}\right)^m.$$

(A) $\frac{4}{27}$

(B) $\frac{16}{27}$

(C) $\frac{32}{27}$

(D) $\frac{8}{27}$

(E) $\frac{2}{27}$

9. What is the smallest number of terms of the convergent series

$$\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^3}$$

that must be summed to be sure that the partial sum is within 10^{-3} of the true sum?

(A) 50
(B) 20
(C) 40
(D) 10
(E) 30

10. The series

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

is

(A) convergent by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) convergent by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(C) divergent by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(D) divergent by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(E) divergent by the test for divergence.

11. We are given two series with positive terms $S_1 = \sum_{k=1}^{\infty} a_k$ and $S_2 = \sum_{k=1}^{\infty} b_k$, and we further know that

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \frac{1}{2} \quad \text{and} \quad \lim_{k\to\infty} \frac{b_k}{a_k} = 2.$$

What can we say about the convergence of these two series based on the given information?

(A) S_1 must be convergent but S_2 could be convergent or divergent.

(B) S_1 must be divergent but S_2 could be convergent or divergent.

(C) We know nothing: S_1 could be convergent or divergent and S_2 could be convergent or divergent.

(D) S_1 and S_2 must both be convergent.

(E) S_1 and S_2 must both be divergent.

12. Which of the following statements is correct about the convergence of the series

$$S_1 = \sum_{k=1}^{\infty} \frac{(-1)^k \sqrt{k}}{k+4} \quad \text{and} \quad S_2 = \sum_{k=1}^{\infty} \frac{(-1)^k 2k^2}{3k^2+1} \quad ?$$

(A) S_1 and S_2 are divergent.

(B) S_1 is absolutely convergent and S_2 is conditionally convergent.

(C) S_1 is conditionally convergent and S_2 is divergent.

(D) S_1 and S_2 are conditionally convergent.

(E) S_1 and S_2 are absolutely convergent.

DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.

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