MA 16200
FINAL EXAM Form 01
December 14, 2023

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below the boxes on the scantron: **01**

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: **01**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros.**

There are **25** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 12:20, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 10:50. If you don't finish before 12:20, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Please do NOT open the exam until instructed to do so.
2. Please obey the instructions and requests by all proctors, TAs, and lecturers.
3. To minimize distractions, you may NOT leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, please put down all writing instruments and remain in your seat. Your TA will collect your scantron and the exam. Once you have turned in your scantron and exam, you are free to leave.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

1. For what value of t is the vector $< 2, t, -1 >$ perpendicular to the vector $< t, 1, 1 >$?

A. $\frac{1}{5}$
B. $\frac{1}{4}$
C. $\frac{1}{3}$
D. $\frac{2}{3}$
E. 1

2. A parallelogram in the $xy-$plane has the vertices $(1, 1)$, $(2, 3)$, $(3, 2)$, and $(4, 4)$. What is the area of that parallelogram?

A. 2
B. 3
C. $\sqrt{2}$
D. $\sqrt{3}$
E. $\frac{3}{2}$

3. Find the volume of the solid obtained by revolving the region under the graph of $y = \frac{1}{x}$, $1 \leq x \leq 2$, about the $x-$axis.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

E. 2π

4. Find the area of the region in the first quadrant of the $xy-$plane bounded above by $y = \frac{\pi}{2}x$ and below by $y = \sin^{-1} x$.

A. $1 - \frac{\pi}{4}$

B. $\frac{1}{2}$

C. $\frac{\pi}{2} - 1$

D. $\frac{\pi^2}{4} - 1$

E. $\frac{1}{4}$

5. Find the volume of the solid obtained by revolving the region in the first quadrant of the $xy-$plane bounded by the curves $y = x$ and $y = x^2$ about the $y-$axis.

A. $\frac{\pi}{2}$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{5}$
E. $\frac{\pi}{6}$

6. Find the length of the curve $y = \dfrac{e^x + e^{-x}}{2}$, $0 \leq x \leq 1$.

A. $e + \frac{1}{e}$
B. $e - \frac{1}{e}$
C. $\frac{e}{2} + \frac{1}{2e}$
D. $\frac{e}{2} - \frac{1}{2e}$
E. $\frac{e}{4} - \frac{1}{4e}$

7. A tank in the shape of a rectangular box has a square base with sides 1 m and a height of 4 m. If it is half full of a liquid that weighs 3 N/m^3, how much work in Nm is needed to pump out all the liquid over the top of the tank?

A. 12

B. 16

C. 18

D. 20

E. 24

8. Find the area of the surface obtained by revolving the graph $y = x^2$, $0 \leq x \leq 1$, about the $y-$axis.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}(5\sqrt{5} - 1)$

C. $\frac{\pi}{3}(5\sqrt{5} - 1)$

D. $\frac{\pi}{4}(5\sqrt{5} - 1)$

E. $\frac{\pi}{6}$

9. Evaluate $\int_1^e x \ln x \, dx$

A. $\frac{e^2-1}{2}$

B. $\frac{e^2+1}{2}$

C. $\frac{e^2-1}{4}$

D. $\frac{e^2+1}{4}$

E. $\frac{e^2}{4}$

10. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cos^3 x \, dx$

A. $\frac{2}{15}$

B. $\frac{4}{15}$

C. $\frac{4}{21}$

D. $\frac{8}{21}$

E. $\frac{3}{10}$

11. Evaluate

$$\int_0^1 \frac{1}{x^2+5x+4}\,dx$$

A. $\ln 2 - \frac{1}{3}\ln 5$
B. $\frac{1}{3}\ln 2 - \frac{1}{3}\ln 5$
C. $\frac{2}{3}\ln 2 - \frac{1}{3}\ln 5$
D. $\frac{1}{3}\ln 2 - \frac{2}{3}\ln 5$
E. $\frac{2}{3}\ln 2 - \frac{2}{3}\ln 5$

12. What substitution should one make to compute

$$\int \frac{1}{\sqrt{-x^2+4x-3}}\,dx$$

A. $x = 2 + \sec\theta$
B. $x = 2 + \tan\theta$
C. $x = \sqrt{3} - \sin\theta$
D. $x = \sqrt{3} + \tan\theta$
E. $x = 2 + \sin\theta$

13. Which of the following improper integrals converge?

I. $\displaystyle\int_1^{\infty} \frac{1}{2+x^2}\,dx$

II. $\displaystyle\int_{-10}^{10} \frac{1}{x^5}\,dx$

III. $\displaystyle\int_{-\infty}^{0} \frac{1}{\sqrt{3-x}}\,dx$

A. I only

B. II only

C. III only

D. I and II only

E. I, II, and III

14. Consider the sequence $\left\{\dfrac{1-2n}{1+3n}\right\}_{n=1}^{\infty}$. Which of the following statements are true?

I. The sequence is increasing.
II. The sequence is bounded.
III. The sequence is convergent.

A. I and II only

B. I and III only

C. II and III only

D. I, II, and III

E. None are true.

15. For which values of p does the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converge? Hint: consider using the Integral Test.

A. $p < 1$

B. $p < 2$

C. $p \neq 0$

D. $p > 1$

E. $p > 2$

16. Determine whether the series converges or diverges. If it is convergent, find its sum.

$$-5 + 2 - \frac{4}{5} + \frac{8}{25} - \frac{16}{125} + - \cdots$$

A. converges to $-\dfrac{2}{5}$

B. converges to 0

C. converges to $-\dfrac{25}{7}$

D. converges to $\dfrac{5}{3}$

E. diverges

17. Which of the following statements is/are true?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{9^n}$ is absolutely convergent

II. $\displaystyle\sum_{n=3}^{\infty} \frac{(-5)^{n+1}}{\sqrt{n}}$ is conditionally convergent

III. the Ratio Test is inconclusive with $\displaystyle\sum_{n=3}^{\infty} \frac{10\sqrt{n}}{1+n^2}$

A. I only

B. II only

C. III only

D. I and III only

E. I, II, and III

18. What is the coefficient of the term containing $(x-1)^2$ in the Taylor series representation of the function $f(x) = \dfrac{-1}{x^2}$ centered at $a = 1$?

A. -3

B. -2

C. -1

D. 1

E. 2

19. If the Maclaurin series of a function $f(x)$ is

$$\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{3n(n+6)},$$

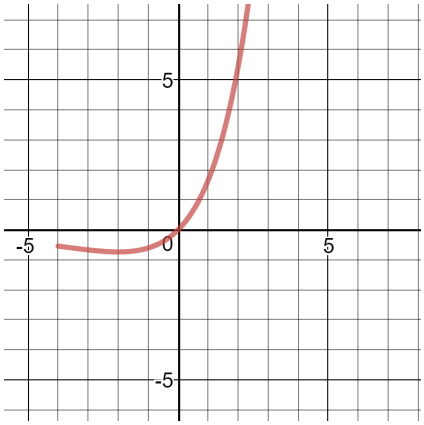
then $f^{(6)}(0)$ (that is, the sixth derivative of $f(x)$ at 0) is equal to

A. $5/3$
B. $-15/6$
C. $10/3$
D. $9/7$
E. $8/5$

20. The first 3 terms of the Maclaurin series of $f(x) = \dfrac{x}{1+x^3}$ are

A. $x + x^4 + x^7$
B. $x - x^4 + x^7$
C. $1 - x^3 + x^6$
D. $1 + x^3 + x^6$
E. $-x - x^4 - x^7$

21. Which of the following could be the Maclaurin series for the function graphed below?



A. $x + \frac{1}{2}x^2 + \frac{1}{8}x^3 + \cdots$
B. $x - \frac{1}{2}x^2 + \frac{1}{8}x^3 - \cdots$
C. $2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + \cdots$
D. $x^2 + \frac{1}{2}x^3 + \frac{1}{8}x^4 + \cdots$
E. $-x^2 - \frac{1}{2}x^3 - \frac{1}{8}x^4 - \cdots$

22. In the power series representation of $\int x^2 e^x \, dx$, what is the term that contains x^6? Note that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

A. $\frac{x^6}{20}$
B. $\frac{x^6}{24}$
C. $\frac{x^6}{28}$
D. $\frac{x^6}{32}$
E. $\frac{x^6}{36}$

23. What is $(\sqrt{2}, \sqrt{6})$ in polar coordinates?

A. $(2\sqrt{2}, \frac{\pi}{6})$

B. $(2\sqrt{2}, \frac{\pi}{3})$

C. $(2\sqrt{2}, \frac{\pi}{2})$

D. $(8, \frac{\pi}{6})$

E. $(8, \frac{\pi}{3})$

24. Liam and Alex are traveling in a spacecraft that follows a path given by the polar equation $r = e^{\theta}$, $0 \leq \theta \leq \frac{\pi}{2}$. What is the distance traveled by the spacecraft?

A. $\sqrt{2}(e^{\frac{\pi}{2}})$

B. $e^{\frac{\pi}{2}} + 1$

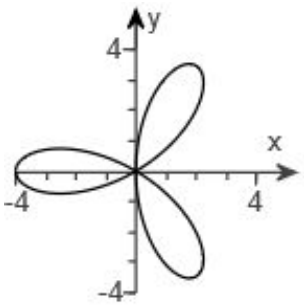
C. $e^{\pi} - \frac{1}{2}$

D. $e^{\frac{\pi}{2}} - 1$

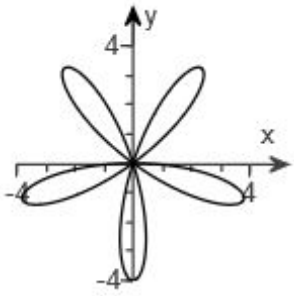
E. $\sqrt{2}(e^{\frac{\pi}{2}} - 1)$

25. Which of the following is the graph of $r = 4\sin(3\theta)$?

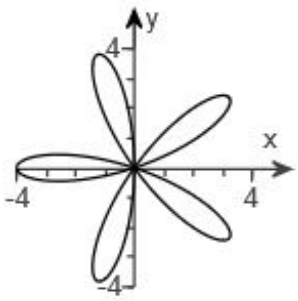
A



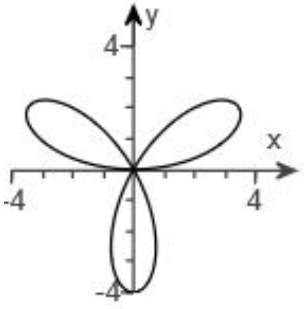
B



C



D



E