MA 16200: Final Examination

Fall 2024, Purdue University

Exam version: 01

Name: _____ PUID #: _____

Exam Instructions:

- Follow these instructions carefully. Failure to do so may result in your exam being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.
- Mark the circle of your recitation section below. Write your name and PUID on the top of this cover page. **DO NOT WRITE ANYTHING ELSE** on this cover page.

	Sec	Time	TA Name
◯	206	7:30AM	Gage Bachmann
◯	109	7:30AM	Lance Daley
◯	904	7:30AM	Luca Mossman
◯	906	7:30AM	Michael Poole
◯	210	7:30AM	Ehan Shah
◯	208	8:30AM	Gage Bachmann
◯	111	8:30AM	Lance Daley
◯	908	8:30AM	Luca Mossman
◯	902	8:30AM	Michael Poole
◯	212	8:30AM	Ehan Shah
◯	224	9:30AM	Niveditha Nerella
◯	216	9:30AM	Claudia Phagan
◯	107	10:30AM	Otto Baier
◯	222	10:30AM	Niveditha Nerella

	Sec	Time	TA Name
◯	214	10:30AM	Claudia Phagan
◯	113	11:30AM	Tausif Ahmed
◯	105	11:30AM	Otto Baier
◯	115	12:30PM	Tausif Ahmed
◯	101	12:30PM	Alexis Cruz Castillo
◯	103	1:30PM	Alexis Cruz Castillo
◯	218	1:30PM	Leo Shen
◯	220	2:30PM	Leo Shen
◯	117	3:30PM	Tifany Burnett
◯	204	3:30PM	Mohamad Mousa
◯	121	3:30PM	Juliet Raginsky
◯	119	4:30PM	Tifany Burnett
◯	202	4:30PM	Mohamad Mousa
◯	123	4:30PM	Juliet Raginsky

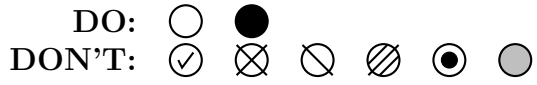
- Detach the final page of formula sheet from the exam, then write your name and PUID on the back cover and the formula sheet. **DO NOT DETACH ANY OTHER PAGES** from the exam booklet.
- This exam consists of 25 questions for a total of 200 points.
- You have exactly two hours to complete the exam.
- Do not open the exam booklet or start writing before the proctor signals the start of the exam.
- Additional pages for scratch work can be found at the end of the booklet.
- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.
- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.
- If you finish the exam before 8:55 pm, you may leave the room after turning in the exam booklet. You may not leave the room before 7:20 pm. If you don't finish before 8:55 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your exam booklet. You must stop working when the proctor signals the end of exam.

Good luck!

DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.

Multiple-choice Instructions:

- For multiple choice questions, fill the circles completely with a **#2 PENCIL** for your answer choices. If you need to change your answer choice, erase the mark completely.



- Only what you marked on this page will be graded for score.
- Partial credit will not be awarded for multiple choice questions.

Mark all your answer choices below:

	A	B	C	D	E
#1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

	A	B	C	D	E
#11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#14	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#15	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#16	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#17	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#18	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#19	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#20	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

	A	B	C	D	E
#21	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#22	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#23	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#24	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
#25	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Exam booklet version:

● 01

○ 02

○ Alternate

○ Enlarged Text

Make no stray marks in the boxed region above. Do not modify the exam booklet version marking.

DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.

Remember to mark all your answer choices on the second cover page!

1. (8 points) What is the radius of convergence of the power series

$$\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^{k^2} x^k \quad ?$$

(A) ∞
(B) $1/e$
(C) 0
(D) e
(E) 1

2. (8 points) Convert the polar coordinates $(r, \theta) = (-3\sqrt{2}, 3\pi/4)$ into Cartesian coordinates.

(A) $(x, y) = (3\sqrt{2}, -3\sqrt{2})$
(B) $(x, y) = (-3, -3)$
(C) $(x, y) = (3, -3)$
(D) $(x, y) = (-3\sqrt{2}, 3\sqrt{2})$
(E) $(x, y) = (-3, 3)$

Remember to mark all your answer choices on the second cover page!

3. (8 points) Evaluate

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot 2^n}$$

(A) $\ln(3/2)$
(B) 1
(C) $\ln(1/2)$
(D) $1/3$
(E) The series is divergent.

4. (8 points) Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{2^n \cdot n^2}$$

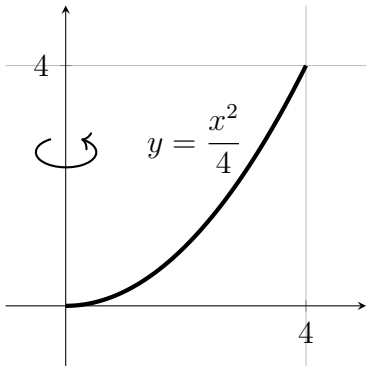
Which of the following applications of convergence tests is/are correct?

(I) The series is convergent by the **comparison test** with $\sum \frac{1}{2^n}$
(II) The series is convergent by the **comparison test** with $\sum \frac{1}{n^2}$
(III) The series is convergent by the **limit comparison test** with $\sum \frac{1}{2^n}$
(IV) The series is convergent by the **limit comparison test** with $\sum \frac{1}{n^2}$

(A) All of (I), (II), (III), and (IV)
(B) (I), (II), and (III)
(C) (III) and (IV)
(D) (I) and (II)
(E) (I) and (III)

Remember to mark all your answer choices on the second cover page!

5. (8 points) A parabolic cereal bowl is 4 cm tall and 4 cm in radius at the rim. The bowl could be considered the result of revolving the parabola $y = \dfrac{x^2}{4}$ for $0 \leq x \leq 4$ around the y-axis. What is the internal volume of the bowl?



(A) $\dfrac{1024}{3}\pi$ cm^3

(B) $\dfrac{16}{3}\pi$ cm^3

(C) 4π cm^3

(D) 80π cm^3

(E) 32π cm^3

6. (8 points) Evaluate

$$\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

(A) $\ln(3)$

(B) $3/2$

(C) 1

(D) 2

(E) The series is divergent.

Remember to mark all your answer choices on the second cover page!

7. (8 points) Three vertices of a triangle are at

$$A = (0, 0, 0), \quad B = (2, -1, 1), \quad C = (3, 0, 4).$$

What is the area of this triangle?

(A) 10
(B) 5
(C) $5\sqrt{2}$
(D) $5\sqrt{2}/2$
(E) 6

8. (8 points) Given a sequence $\{a_n\}_{n=1}^{\infty}$ whose terms are all positive, which of the following statements is/are true?

(I) If $\{a_n\}_{n=1}^{\infty}$ is convergent, then $\left\{\dfrac{a_n}{n}\right\}_{n=1}^{\infty}$ is convergent.

(II) If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, then $\{a_n\}_{n=1}^{\infty}$ is convergent.

(III) If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, then $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n}$ is convergent.

(A) (II) only
(B) (I) and (II)
(C) All of (I), (II), and (III)
(D) (I) and (III)
(E) (I) only

Remember to mark all your answer choices on the second cover page!

9. (8 points) Find the value of A in the equation

$$\int_0^1 x^2 e^{-2x}\,dx = A + \int_0^1 xe^{-2x}\,dx$$

(A) $\dfrac{2}{e^2}$

(B) $-\dfrac{1}{2e^2}$

(C) $\dfrac{1}{2e^2}$

(D) $-\dfrac{1}{e^2}$

(E) $-\dfrac{2}{e^2}$

10. (8 points) Three vertices of a triangle are at

$$A = (1, 2, 3), \quad B = (0, 2, 4), \quad C = (2, 4, 1).$$

What is the angle at the vertex A?

(A) $\pi/6$

(B) $3\pi/4$

(C) $2\pi/3$

(D) $\pi/4$

(E) $\pi/3$

Remember to mark all your answer choices on the second cover page!

11. (8 points) Which of the following polar coordinates correspond(s) to the point $(x, y) = (-\sqrt{3}, 3)$ in Cartesian coordinates?

(I) $P_1(r, \theta) = (-2\sqrt{3}, -2\pi/3)$
(II) $P_2(r, \theta) = (2\sqrt{3}, -4\pi/3)$
(III) $P_3(r, \theta) = (6, 2\pi/3)$

(A) P_1 only
(B) P_2 only
(C) P_3 only
(D) P_1 and P_2
(E) All of P_1, P_2, and P_3

12. (8 points) What is the result of approximating the value of the integral

$$\int_0^1 e^x \, dx$$

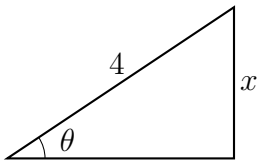
using the second-order Taylor polynomial of e^x centered at the origin?

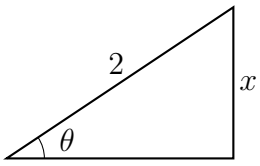
(A) 2/3
(B) 2
(C) 5/3
(D) 3/2
(E) 5/2

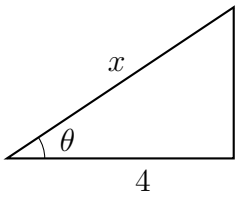
Remember to mark all your answer choices on the second cover page!

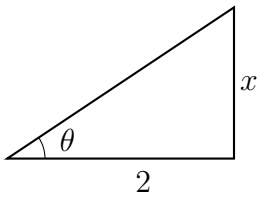
13. (8 points) Which one of the following right triangles is most suitable for a trigonometric substitution on the integral

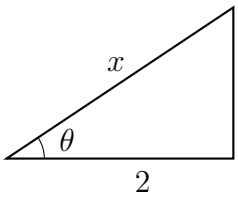
$$\int \frac{1}{(4-x^2)^{3/2}}\,dx \quad ?$$

(A) 

(C) 

(E) 

(B) 

(D) 

14. (8 points) Which one of the following polar curves is a horizontal line?

(A) $r = \cos(\theta)$

(B) $r = \theta$

(C) $r = 1$

(D) $r = \sec(\theta)$

(E) $r = \csc(\theta)$

Remember to mark all your answer choices on the second cover page!

15. (8 points) A cylindrical tank has height 4 m and base radius 3 m. The tank is completely filled with water. Compute the work required to pump all water out through the top rim of the tank.

    [Use $\rho = 1000$ kg/m^3 for the density of water, and $g = 10$ m/s^2 for gravitational acceleration.]

    (A) 240000π J

    (B) 480000π J

    (C) 1440000π J

    (D) 720000π J

    (E) 360000π J

16. (8 points) Given two points $P = (2, -1, 2)$ and $Q = (-2, 0, -6)$, find a unit vector in the $\overrightarrow{PQ}$ direction.

    (A) $\left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$

    (B) $\left\langle \frac{4}{9}, -\frac{1}{9}, \frac{8}{9} \right\rangle$

    (C) $\left\langle \frac{4}{9}, \frac{1}{9}, \frac{8}{9} \right\rangle$

    (D) $\left\langle -\frac{4}{9}, \frac{1}{9}, -\frac{8}{9} \right\rangle$

    (E) $\left\langle -\frac{1}{\sqrt{10}}, 0, -\frac{3}{\sqrt{10}} \right\rangle$

Remember to mark all your answer choices on the second cover page!

17. (8 points) Let $f(x) = \sum_{k=1}^{\infty} \frac{1}{k^2} x^k$ be a function defined by power series. Evaluate

$$\lim_{x \to 0} \frac{f(x) + f(-x)}{x^2}$$

(A) 1/4
(B) 1/2
(C) 0
(D) 2
(E) The limit does not exist.

18. (8 points) Find the arc length of the polar curve $r = \sin(\theta) - \cos(\theta)$, where $0 \leq \theta \leq \pi$.

(A) 2π
(B) $2\sqrt{2}\pi$
(C) $\sqrt{2}\pi$
(D) $\sqrt{2}\pi/2$
(E) 4

Remember to mark all your answer choices on the second cover page!

19. (8 points) If $\vec{u} = \langle 1, 3\rangle$, $\vec{v} = \langle 0, 2\rangle$, and $\vec{w} = \langle 3, 4\rangle$, evaluate

$$(\vec{u} - 2\vec{v}) \cdot \vec{w}$$

(A) 7
(B) $\langle 3, -4\rangle$
(C) -1
(D) $5\sqrt{2}$
(E) $\langle 0, 0, 7\rangle$

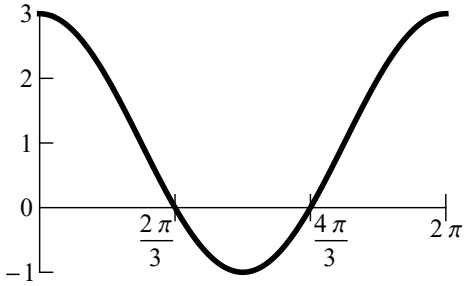
20. (8 points) Given vectors $\vec{v}$ and $\vec{w}$ in 3D, which of the following statements is/are true?

(I) If $\vec{v} \cdot \vec{w} = 0$, then $|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}|$.
(II) If $\vec{v} \times \vec{w} = \vec{0}$, then $|\vec{v} \cdot \vec{w}| = |\vec{v}||\vec{w}|$.
(III) If $|\vec{v}| = |\vec{w}| = 1$, then $\vec{v} \cdot \vec{w} \leq 1$.

(A) (III) only
(B) All of (I), (II), and (III)
(C) (I) only
(D) (I) and (II)
(E) (II) only

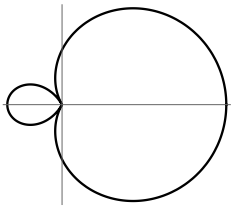
Remember to mark all your answer choices on the second cover page!

21. (8 points) Let $f(x)$ be a function with the following graph in Cartesian coordinates.

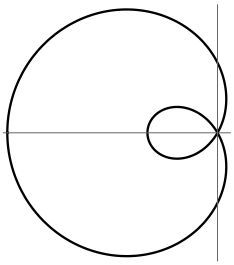


Which one of the following shows the polar curve $r = f(\theta)$ relative to the coordinate axes?

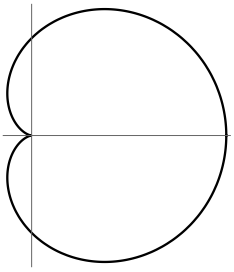
(A)



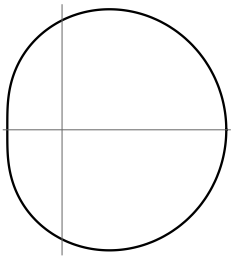
(B)



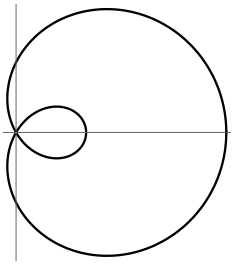
(C)



(D)



(E)



Remember to mark all your answer choices on the second cover page!

22. (8 points) Evaluate

$$\int_1^\infty \frac{x-1}{x^4}\,dx$$

(A) The integral is divergent.
(B) 5/6
(C) 1/6
(D) 1/2
(E) 1/3

23. (8 points) Among the vectors

$$\vec{a} = \langle -1, -1, -1\rangle, \quad \vec{b} = \langle 1, 2, -3\rangle, \quad \vec{c} = \langle -1, 0, 1\rangle, \quad \vec{d} = \langle 0, 0, 0\rangle,$$

which one(s) is/are orthogonal to the vector $\langle 1, 1, 1\rangle$?

(A) All of $\vec{a}$, $\vec{b}$, $\vec{c}$, and $\vec{d}$
(B) $\vec{a}$ and $\vec{b}$
(C) $\vec{b}$, $\vec{c}$, and $\vec{d}$
(D) $\vec{b}$ and $\vec{c}$
(E) $\vec{a}$ only

Remember to mark all your answer choices on the second cover page!

24. (8 points) Consider the series

$$S = \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n + 7}.$$

By the error estimate for alternating series, how many terms are needed for the partial sum S_n to get within 0.01 of the true sum S?

(A) 10
(B) 9
(C) 8
(D) 7
(E) 6

25. (8 points) Find the area of the region enclosed by the polar curve $r = 2 + \cos(\theta)$, $0 \leq \theta \leq 2\pi$.

(A) $9\pi/2$
(B) $17\pi/2$
(C) 8π
(D) 4π
(E) 9π

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Remember to mark all your answer choices on the second cover page!
DO NOT DETACH THIS PAGE FROM THE EXAM BOOKLET.
Write down your name and PUID on this page.

Name: _____ PUID #: _____

Do not write anything else on this page.

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Do not write anything else on this page.

Remember to mark all your answer choices on the second cover page!
DETACH THIS FORMULA SHEET FROM THE EXAM BOOKLET.
Write down your name and PUID on the back.

Do not write anything on this page.

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Remember to mark all your answer choices on the second cover page!
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Name: _____ PUID #: _____

Common Maclaurin series:

$$\begin{aligned}
e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots &&= \sum_{k=0}^{\infty} \frac{x^k}{k!}, && \text{for } -\infty < x < \infty \\
\sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, && \text{for } -\infty < x < \infty \\
\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, && \text{for } -\infty < x < \infty \\
\ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots &&= \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, && \text{for } -1 < x \leq 1 \\
\arctan(x) &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots &&= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, && \text{for } -1 \leq x \leq 1
\end{aligned}$$

Common trigonometric identities:

$$\begin{aligned}
&\cos(x)^2 + \sin(x)^2 = 1 \\
&\sec(x)^2 - \tan(x)^2 = 1 \\
&\sin(2x) = 2\cos(x)\sin(x) \\
&\cos(2x) = \cos(x)^2 - \sin(x)^2 \\
&\cos(2x) = 2\cos(x)^2 - 1 \\
&\cos(2x) = 1 - 2\sin(x)^2
\end{aligned}$$

Special values of trigonometric functions:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	π
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	0
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	-1
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—	$-\sqrt{3}$	-1	0