MA 16500
FINAL EXAM INSTRUCTIONS
VERSION 01
December 10, 2024

Your name ____________________ Your TA's name ____________________

Student ID # _______________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. **Sign the scantron sheet.**

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. While mark all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 25 questions, each of which is worth 8 points. The maximum possible score is

$$25 \textbf{ questions } \times 8 \textbf{ points } = 200 \textbf{ points}.$$

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 8:55, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 8:55, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. (8 points) Compute the following integral using the geometric interpretation.

$$\int_3^5 \sqrt{16-(x-3)^2}\;dx.$$

A. $\dfrac{4\pi}{3}+\dfrac{1}{2}$

B. $\dfrac{4\pi}{3}+\dfrac{\sqrt{3}}{2}$

C. $\dfrac{4\pi}{3}+2\sqrt{3}$

D. $2\pi+\sqrt{3}$

E. 2π

2. (8 points) Set

$$F(x) = \int_{\sqrt{x}}^{x^2} \frac{e^t}{t}\, dt.$$

Find the formula for $F'(x)$.

A. $\dfrac{e^{x^2} - e^{\sqrt{x}}}{2x}$

B. $\dfrac{4e^{x^2} - e^{\sqrt{x}}}{2x}$

C. $\dfrac{4e^{x} - e^{\sqrt{x}}}{x}$

D. $\dfrac{e^{x^2} - e^{\sqrt{x}}}{x}$

E. $\dfrac{e^{x^2 - \sqrt{x}}}{2x}$

3. (8 points) Compute the following limit

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{n+4k}\right).$$

HINT: Identify the sum as an approximation for a defnite integral (i.e., a Riemann Sum).

Then compute the limit as the definite integral.

Use the formula below if necessary:

$$\sum_{k=1}^{n}\frac{1}{n+4k}=\sum_{k=1}^{n}\frac{1}{1+4(k/n)}\cdot\frac{1}{n}.$$

A. $\dfrac{\ln 2}{4}$

B. $\dfrac{\ln 3}{4}$

C. $\dfrac{\ln 5}{4}$

D. $\dfrac{\pi}{4}$

E. $\dfrac{\pi}{2}$

4. (8 points) The population of a town with population of $90,000$ in yera 2016 grows at a rate of 2.4 % per year.

After how many years will the population reach $120,000$?

A. $\dfrac{\ln\left(\frac{4}{3}\right)}{\ln(0.024)}$ years

B. $\dfrac{\ln 0.024}{\ln\left(\frac{4}{3}\right)}$ years

C. $\dfrac{\ln\left(\frac{4}{3}\right)}{\ln(1.024)}$ years

D. $\dfrac{2\ln 1.5}{\ln 3}$ years

E. $\dfrac{\ln\left(\frac{3}{4}\right)}{\ln(1.024)}$ years

5. (8 points) Compute the following integral

$$\int_0^{\ln\sqrt{3}} \frac{e^x}{1+e^{2x}}\,dx.$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{12}$

D. $\ln(1+2\sqrt{3})$

E. $\ln 2$

6. (8 points) A 15-foot ladder is leaning against a vertical wall and its bottom is being pushed toward the wall at the rate of 3 ft/sec.

At what rate is the acute angle θ between the ladder and the ground changing when the acute angle the ladder makes with the ground is $\pi/4$?

A. $\dfrac{2\sqrt{2}}{15}$ rad/sec

B. $\dfrac{\sqrt{2}}{3}$ rad/sec

C. $\dfrac{\sqrt{2}}{2}$ rad/sec

D. $\dfrac{\sqrt{2}}{5}$ rad/sec

E. $\dfrac{3\sqrt{2}}{5}$ rad/sec

7. (8 points) Sand is being dumped from a conveyor belt at the rate of 30 ft^3/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal.

How fast is the height of the pile increasing when the pile is 10 ft high ?

HINT: The volume of a circular cone whose radius at the bottom is r and whose height is h is given by:

$$V = \frac{1}{3}\pi r^2 h.$$

A. $\frac{6}{5\pi}$ ft/min

B. $\frac{3}{5\pi}$ ft/min

C. $\frac{3\pi}{2}$ ft/min

D. 3π ft/min

E. $\frac{3}{10\pi}$ ft/min

8. (8 points) Compute the slope of the tangent line to the curve

$$y = x^{\cos(x)}$$

at the point $(\pi, 1/\pi)$.

A. $\dfrac{1}{\pi}$

B. $\dfrac{-1}{\pi^2}$

C. $\dfrac{\ln(\pi)}{\pi}$

D. $\dfrac{\pi \ln(\pi) - 1}{\pi^2}$

E. $\dfrac{-1}{\pi}$

9. (8 points) What is the smallest possible area of a triangle formed by the coordinate axes and a line tangent to the ellipse

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$$

in the first quadrant ?

A. 1

B. 3

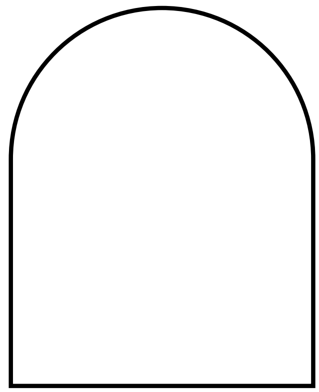
C. 5

D. 15

E. 21

10. (8 points) A *Norman window* is a window in the shape of a rectangle surmounted by a semicircle whose diameter equals the base of the rectangle.

What is the largest possible area of a Norman window with perimeter 4 m ?



A. $\dfrac{4}{4+\pi}$ m

B. $\dfrac{8}{4+\pi}$ m

C. $\dfrac{4}{4-\pi}$ m

D. $4+\dfrac{\pi}{2}$ m

E. 2 m

CORRECTION: What is the radius of the semicircle when the Norman window has the largest possible area with perimeter 4 m ?

11. (8 points) If we use the linear approximation of the function $f(x) = \sqrt[4]{16+x}$ at $a = 0$, then the estimate of $\sqrt[4]{16.32}$ is given by:

A. 2.32

B. 2.51

C. 2.01

D. 2.04

E. 2.08

12. (8 points) Find the real numbers a and b so that the following function becomes continuous over $(-\infty, \infty)$:

$$f(x) = \begin{cases} a & \text{if} \quad x \leq 1 \\ \dfrac{(\sqrt{x^2+8} - b)}{x^2 - 1} & \text{if} \quad x > 1. \end{cases}$$

A. $a = 1/6, b = 3$

B. $a = 1, b = 3$

C. $a = -1/6, b = -3$

D. $a = -1, b = -3$

E. No matter how you choose a and b, the function cannot be continuous over $(-\infty, \infty)$.

13. (8 points) Given the following function with the prescribed domain, find the formula and the domain of its inverse function.

$$f(x) = \frac{6x-1}{2x+1} \text{ defined over } (1, \infty).$$

A. $f^{-1}(x) = \dfrac{x+1}{2x-6}$. Domain of $f^{-1} = (-\infty, 3)$

B. $f^{-1}(x) = \dfrac{-x-1}{2x-6}$. Domain of $f^{-1} = (-\infty, 3)$

C. $f^{-1}(x) = \dfrac{-x-1}{2x-6}$. Domain of $f^{-1} = (5/3, 3)$

D. $f^{-1}(x) = \dfrac{-x-1}{2x+6}$. Domain of $f^{-1} = (5/3, 3)$

E. $f^{-1}(x) = \dfrac{-x-1}{2x-6}$. Domain of $f^{-1} = (5/3, \infty)$

14. (8 points) Find the slope of the tangent to the curve given by the equation

$$x^2 \sin(2y) = y^2 \cos(2x)$$

at the point $(\pi/4, \pi/2)$.

A. 2

B. 4

C. 0

D. $2/\pi$

E. $4/\pi$

15. (8 points) Find all the values x on the interval $[0, 2\pi]$ satisfying the equation

$$\sin(2x) - \sin x = 0.$$

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

B. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

C. $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

D. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

E. $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}$

16. (8 points) Consider the function

$$f(x) = (x+3)^6(x-5)^4.$$

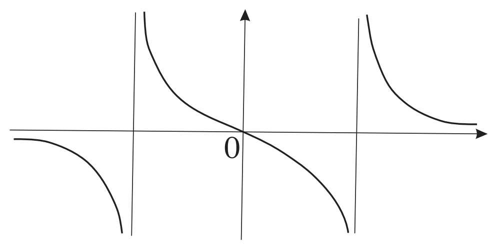
Find the value(s) of x whrere the function takes its
(a) Local Max, and (b) Local Min.

A. (a) Local Max at $x = -3$, and (b) Local Min at $x = 5$.
B. (a) Local Max at $x = 9/5$, and (b) Local Min at $x = 5$.
C. (a) Local Max at $x = 9/5$, and (b) Local Min at $x = -3$.
D. (a) Local Max at $x = -3, 5$, and (b) Local Min at $x = 9/5$.
E. (a) Local Max at $x = 9/5$, and (b) Local Min at $-3, 5$.

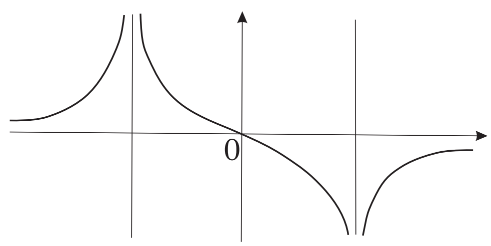
17. (8 points) Which of the following best describes the graph of the function

$$y = f(x) = \frac{x^3}{1-x^2} \ ?$$

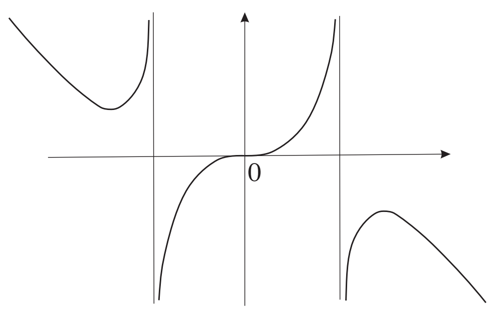
A.



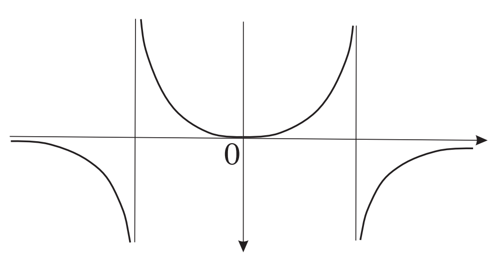
B.



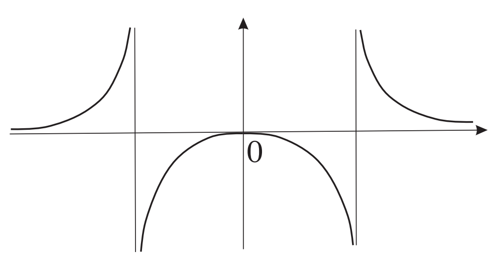
C.



D.



E.



18. (8 points) Compute the following limit

$$\lim_{x \to 0} \frac{\tan(x) - x}{x^3}.$$

A. 2

B. 1

C. 1/2

D. 1/3

E. 1/4

19. (8 points) Compute the following limit

$$\lim_{x \to 0^+} (1 + \sin(4x))^{\cot(3x)}.$$

A. 1

B. e

C. $e^{3/4}$

D. $e^{4/3}$

E. e^{12}

20. (8 points) Compute the following limit

$$\lim_{x \to \infty} \sin(x) \cdot \tan\left(\frac{3}{x}\right).$$

A. 3

B. 2

C. 1

D. $\pi/2$

E. 0

21. (8 points) Five students, Aiden, Bruki, Charles, David, and Ethan, are arguing about the exact value(s) of

$$\begin{cases} f(7) &= \tan^{-1}(7) - \tan^{-1}\left(-\frac{1}{7}\right), \text{ and} \\ f(-7) &= \tan^{-1}(-7) - \tan^{-1}\left(\frac{1}{7}\right), \end{cases}$$

where

$$f(x) = \tan^{-1}(x) - \tan^{-1}\left(-\frac{1}{x}\right).$$

Everybody agrees that $f'(x) = 0$.

But they cannot agree on its implication and how to use it to compute the exact value(s).

Choose the right statement from the comments of the five students below:

A. Aiden says: f is a constant function. Therefore, we have

$$f(7) = f(-7) = f(1) = \tan^{-1}(1) - \tan^{-1}(-1) = \pi/2.$$

B. Bruki says:f is a constant function over $(-\infty, 0)$ and $(0, \infty)$ separately. Therefore, we have

$$f(7) = f(1) = \tan^{-1}(1) - \tan^{-1}(-1) = \pi/2$$

and

$$f(-7) = f(-1) = \tan^{-1}(-1) - \tan^{-1}(1) = -\pi/2.$$

C. Charles says: f is a constant function over $(-\infty, \infty)$. Therefore, we have

$$f(7) = f(-7) = f(0) = \tan^{-1}(0) - \tan^{-1}(-\infty) = \pi/2.$$

D. David says: f is a constant function over $(-\infty, 0)$ and $(0, \infty)$ separately. Therefore, we have

$$f(7) = f(1) = \tan^{-1}(1) - \tan^{-1}(-1) = -\pi/2$$

and

$$f(-7) = f(-1) = \tan^{-1}(-1) - \tan^{-1}(1) = \pi/2.$$

E. Ethan says: f is a constant function. Therefore, we have

$$f(7) = f(-7) = f(-1) = \tan^{-1}(-1) - \tan^{-1}(1) = -\pi/2.$$

22. (8 points) Compute the absolute maximum and absolute minimum of the function $f(x) = 2x^3 + 3x^2 - 12x - 9$ on the interval $[-3, 3]$.

A. Max = 36, Min = 0

B. Max = 36, Min = -16

C. Max = 11, Min = -16

D. Max = 11, Min = 0

E. Max = 36, Min = -9

23. (8 points) Consider the following function

$$f(\theta) = \ln|\sec(3\theta) + \tan(3\theta)|.$$

Find $f'(\pi/3)$.

A. -1

B. 1

C. -3

D. 3

E. 0

24. (8 points) Consider the following picewise defined function

$$f(x) = \begin{cases} 3x & \text{if } \ x < 1 \\ x^3 + 2 & \text{if } \ x \geq 1. \end{cases}$$

Find the correct statement from below about

(a) continuity and (b) differentiability

of the function f at $x = 1$.

A. (a) continuous (b) differentiable
B. (a) NOT continuous (b) differentiable
C. (a) continuous (b) NOT differentiable
D. (a) NOT continuous (b) NOT differentiable
E. (a) continuous (b) one cannot detemine the differentiability from the given information.

25. (8 points) The position s of a particle is given as a function of time t in seconds by the formula

$$s = f(t) = 2t^3 - 9t^2 + 12t$$

Find the total distance traveled during the first 4 seconds.

A. 25

B. 28

C. 30

D. 32

E. 34