MA 16600
EXAM 1 INSTRUCTIONS
VERSION 01
February 6, 2024

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is Green and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While you will mark all your answers on the scantron sheet, you should show your work in the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 12 questions, each worths 10 point. You get 4 points for participating in the exam. The maximum possible score is

12 questions $\times$ 8 points $+$ 4 free points $=$ 100 points.

9. NO calculators, electronic devices, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students who are more than 20 min late will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. (8 Points) Find the center and radius of the sphere defined by the following equation

$$x^2 + 2x + y^2 - 4y + z^2 + 12z = 8$$

A. $(-1, 2, -6)$ and Radius 7

B. Center $(-1, 2, -6)$ and Radius $\sqrt{8}$

C. Center $(1, -2, 6)$ and Radius $\sqrt{8}$

D. Center $(-1, 2, -6)$ and Radius $\sqrt{7}$

E. Center $(1, -2, 6)$ and Radius 7

2. (8 Points) Given $|\vec{u}| = 3$, $|\vec{v}| = 5$ and the angle between $\vec{u}$ and $\vec{v}$ is $\pi/3$, find $|\vec{u} + \vec{v}|$.
(Hint: You may use $|\vec{u} + \vec{v}|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$.)

A. 3

B. 5

C. 7

D. 9

E. 11

3. (8 points) Find $\text{Proj}_{\overrightarrow{v}}\overrightarrow{u}$ (the orthogonal projection of $\overrightarrow{u}$ onto $\overrightarrow{v}$), where $\overrightarrow{u} = \langle 2, 1, 1\rangle$ and $\overrightarrow{v} = \langle 3, 0, 4\rangle$.

A. $2\overrightarrow{u}$

B. $2\overrightarrow{v}$

C. $\dfrac{1}{25}\overrightarrow{v}$

D. $\dfrac{2}{5}\overrightarrow{u}$

E. $\dfrac{2}{5}\overrightarrow{v}$

4. (8 Points) The area of a triangle with vertices $(2, 1, 1)$, $(2, 2, 3)$ and $(1, 2, 2)$ is

A. $\sqrt{6}$

B. $\sqrt{6}/2$

C. $\sqrt{15}/2$

D. $\sqrt{15}$

E. 2

5. (8 Points) Find the area of the region bounded by the curves $y = x^2$ and $y = |x|$.

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. 0

D. $\frac{-1}{3}$

E. 2

6. (8 Points) Consider the region enclosed by the line $y = x$ and $y = x^3$ in the first quadrant. Find the formula for the volume of the solid obtained by rotating the region about the x-axis by the Washer method.

A. $\int_0^1 \pi(x^2 - x^6)dx$

B. $\int_0^1 \pi(x^2 - x^9)dx$

C. $\int_0^1 2\pi(y^{2/3} - y^2)dy$

D. $\int_0^1 \pi(x - x^3)dx$

E. $\int_0^1 2\pi y(y - y^{1/3})dy$

7. (8 Points) Which of the following integral gives the volume of the solid obtained by rotating the region enclosed by the curves $y = x$ and $y = x^2$ about the line $y = 2$ by the Shell method.

A. $\int_0^1 2\pi(2 + y)(x - x^2)dy$

B. $\int_0^1 2\pi(2 - y)(y - y^2)dy$

C. $\int_0^1 2\pi(2 - y)(\sqrt{y} - y)dy$

D. $\int_0^1 2\pi(2 - x)(x^2 - x)dx$

E. $\int_0^1 2\pi(2 + x)(\sqrt{x} - x)dx$

8. (8 Points) If the work required to stretch a spring 4 ft beyond natural length is 32 ft-lb, how much work is needed to stretch it 3 inches beyond its natural length?

A. 24 ft-lb

B. 10 ft-lb

C. 9 ft-lb

D. 8 ft-lb

E. 18 ft-lb

9. (8 Points) A tank has the shape obtained by rotating the curve $y = x^3 (0 \leq x \leq 2)$ about the y-axis with unit in foot. The tank is full of liquid, of which each cubic foot weighs 3 lb (pounds). The work required to empty the tank by pumping out all the liquid from the top of the tank is given by the formula

A. $\pi \int_0^2 x^2 dx$ ft-lb.

B. $\pi \int_0^{16} x^2 dy$ ft-lb.

C. $3\pi \int_0^2 x^2 (2 - x) dx$ ft-lb.

D. $3\pi \int_0^2 y^2 (16 - y) dy$ ft-lb.

E. $3\pi \int_0^8 y^{\frac{2}{3}} (8 - y) dy$ ft-lb.

10. (8 Points) Find the length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x} \quad \text{on } [1, 2].$$

A. $\frac{e}{4}$

B. $\frac{103}{49}$

C. $\frac{59}{24}$

D. $\frac{12}{7}$

E. $\frac{37}{14}$

11. (8 Points) Find the volume of the solid S described below:

(a) The base of S is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$.

(b) The cross section perpendicular to the base and and parallel to the y-axis are squares with one side on the base.

A. $\dfrac{2\pi}{3}$

B. $\dfrac{24}{5}$

C. $\dfrac{52\pi}{5}$

D. $\dfrac{64}{15}$

E. $\dfrac{\pi}{3}$

12. (8 Points)Evaluate $\int_0^{\pi/2} x^2 \sin(x) dx$.

A. $\pi/4$

B. $\pi/1 - 1$

C. $-\pi + 1$

D. $\pi - 2$

E. $\pi/2 - \pi/4$