

MA 16600
FINAL EXAM VERSION 01
May 3, 2024

INSTRUCTIONS

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).
2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.
3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.
4. Fill in your **NAME** and **PURDUE ID NUMBER**, and blacken in the appropriate spaces. Put 00 at the front of PUID to make it a 10 digit number, and then fill it in.
5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.
6. **Sign the scantron sheet.**
7. Blacken your choice of the correct answer in the space provided for each of the questions 1–25. While mark all your answers on the scantron sheet, you should **show your work** on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.
8. There are 25 questions, each of which is worth 8 points. The maximum possible score is $25 \text{ questions} \times 8 \text{ points} = 200 \text{ points}$.
9. **NO calculators, electronic device, books, or papers are allowed.** Use the back of the test pages for scrap paper.
10. After you finish the exam, **turn in BOTH the scantron sheet and the exam booklet.**
11. If you finish the exam before 2:55 PM, you may leave the room after turning in the scantron sheet and the exam booklet. **If you don't finish before 2:55 PM, you should REMAIN SEATED** until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Recitation Section numbers by TA:

7:30 am Lecture		
TA name	Time & Location	Section #
Yannis Yicong Wu-Yip	4:30pm in PHYS 110	0208
Yannis Yicong Wu-Yip	3:30pm in PHYS 110	0215
Yannis Yicong Wu-Yip	2:30pm in PHYS 110	0222
Deanna Marie Wightman	3:30pm in PHYS 202	0229
Deanna Marie Wightman	2:30pm in PHYS 202	0236
Deanna Marie Wightman	4:30pm in PHYS 202	0243
Barath Balasubramanian Ramakrishna	9:30pm in PHYS 333	0250
Barath Balasubramanian Ramakrishna	8:30am in PHYS 333	0257
Barath Balasubramanian Ramakrishna	10:30am in PHYS 333	0264
Mei Lam Chan	1:30pm in Schm 307	0296
Mei Lam Chan	2:30pm in Schm 307	0297
Mei Lam Chan	12:30pm in Schm 307	0298

1:30pm Lecture		
TA name	Time & Location	Section #
Shengwei Qiu	2:30pm in PHYS 333	0271
Shengwei Qiu	1:30pm in PHYS 333	0278
Shengwei Qiu	12:30pm in PHYS 333	0285
Giancarlo Thomas Arcese	2:30pm in HAMP 2101	0287
Giancarlo Thomas Arcese	1:30pm in HAMP 2101	0288
Giancarlo Thomas Arcese	12:30pm in HAMP 2101	0289
Jiarui Huang	10:30am in PHYS 202	0290
Jiarui Huang	12:30pm in PHYS 202	0291
Jiarui Huang	11:30am in PHYS 202	0292
Asini Anuradhika Konpola	9:30am in PHYS 202	0293
Asini Anuradhika Konpola	7:30am in PHYS 202	0294
Asini Anuradhika Konpola	8:30am in PHYS 202	0295

Questions

1. (8 points) Find the slope of the tangent line to the curve $r = 2 \cos(2\theta)$ at $(1, \frac{\pi}{6})$.

A. $\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}}{7}$

C. $\frac{1}{2}$

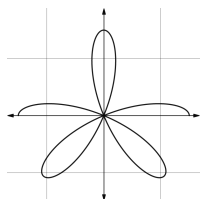
D. Does not exist.

E. 0

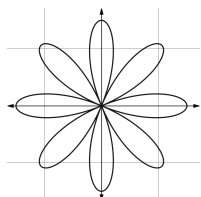
2. (8 points) Choose the picture which best depicts the curve defined by the following equation in polar coordinates

$$r = 3 \cos(4\theta), \quad 0 \leq \theta \leq \pi.$$

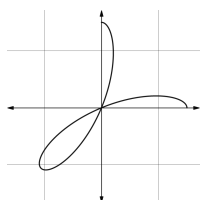
A.



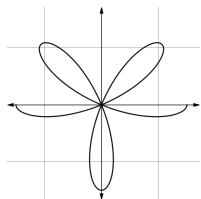
B.



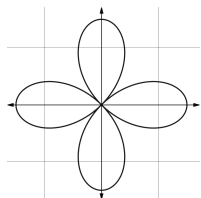
C.



D.



E.



3. (8 points) The graph of the polar equation $r = \frac{3}{\sin \theta}$ is

- A. a vertical line.
- B. a horizontal line .
- C. a circle centered at a point on y axis.
- D. a circle centered at a point on x axis.
- E. a cardioid.

4. (8 points) Find the area between the curves defined by the polar equations

$$r = \theta^2 + 1 \quad \& \quad r = 1 \quad 0 \leq \theta \leq 1.$$

A. $13/30$

B. $1/3$

C. $4/15$

D. $2/3$

E. $\pi/30$

5. (8 points) The equation of a plane is given by:

$$3x - 2y + 13z = 10.$$

Find the value of a such that the vector $\langle a, 4, 2 \rangle$ is parallel to the plane.

$a =$

A. 4

B. -6

C. 3

D. $-\frac{2}{3}$

E. There is no such value of a .

6. (8 points) Find the volume of the parallelepiped, where the following three vectors correspond to the three sides of the parallelepiped are given by

$$\begin{cases} \vec{OA} = \langle 2, 1, 1 \rangle \\ \vec{OB} = \langle 0, 0, 1 \rangle \\ \vec{OC} = \langle -1, 0, 0 \rangle \end{cases}$$

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

7. (8 points) Find the area of the region bounded by the curves $x = 4 + 6y^2$ and $x = 10$.

- A. $\frac{8}{3}$
- B. $\frac{16}{3}$
- C. 8
- D. 16
- E. 3

8. (8 Points) A spherical tank 10 feet in diameter is half full with water (density 62.5 lb/ft³). The work required to pump all of the water in the tank to the top of the tank is given by the following integral. (Let the origin be the center of the tank with the y -axis pointing upward).

A. $\int_{-5}^0 62.5(5 - y)\pi(25 - y^2) dy.$

B. $\int_{-5}^0 62.5(5 - y)\pi\sqrt{25 - y^2} dy.$

C. $\int_0^5 62.5(5 - y)\pi(5 - y^2) dy.$

D. $\int_0^5 62.5(5 + y)\pi(25 - y^2) dy.$

E. $\int_{-5}^0 62.5(5 + y)\pi\sqrt{25 - y^2} dy.$

9. (8 points) Find the volume of the solid generated by revolving about the y-axis the region bounded by

$$y = 0, \quad y = \frac{1}{x + x^3}, \quad x = 1, \quad \text{and} \quad x = 2.$$

- A. $2\pi \arctan(2) - \frac{\pi^2}{2}$
- B. $2\pi \ln(2)$
- C. $2\pi \arctan(2) - \frac{\pi}{4}$
- D. $\frac{2\pi}{3} \ln(9)$
- E. $2\pi \arctan(2)$

10. (8 points) The graph of $f(x) = \frac{1}{4x} + \frac{x^3}{3}$ on the interval $[1, 4]$ is revolved about the x -axis.

Find the right formula that describes the area of the surface generated.

- A. $\int_1^4 2\pi\left(\frac{1}{4x} + \frac{x^3}{3}\right)^2 dx$
- B. $\int_1^4 2\pi(x)\left(\frac{1}{4x^2} + x^2\right) dx$
- C. $\int_1^4 2\pi\left(\frac{1}{4x} + \frac{x^3}{3}\right)\left(\frac{1}{4x^2} + x^2\right) dx$
- D. $\int_1^4 2\pi\left(\frac{1}{4x} + \frac{x^3}{3}\right)\sqrt{\frac{1}{4x^2} + x^2} dx$
- E. $\int_1^4 \pi\left(\frac{1}{4x^2} + x^2\right)^2\sqrt{\frac{1}{4x^2} + x^2} dx$

11. (8 points) Compute $\int_0^{\frac{\pi}{2}} x^2 \cos(x) dx$.

A. $2 - \frac{\pi^2}{4}$

B. $\frac{\pi^2}{4}$

C. $\frac{\pi^2}{4} - 2$

D. $\frac{\pi}{4} + 2$

E. 2

12. (8 points) Compute $\int_3^4 \frac{2}{x^2-3x+2} dx$.

A. $\ln 4 + 2 \ln 3$

B. $2 \ln 2 - \ln 9$

C. $2 \ln 4 - 2 \ln 2$

D. $4 \ln 2 - 2 \ln 3$

E. $2 \ln 5 + 6 \ln 3$

13. (8 points) Write out the form of the partial fraction decomposition of the function

$$\frac{x^3 + x^2 + 10}{(x - 1)^2(x^4 - 1)}.$$

- A. $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{(x - 1)^3} + \frac{D}{x + 1} + \frac{Ex + F}{x^2 + 1}$
- B. $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{x^2 - 1} + \frac{Ex + F}{x^2 + 1}$
- C. $\frac{Ax + B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 1} + \frac{Ex^3 + Fx^2 + Gx + H}{x^4 - 1}$
- D. $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 1} + \frac{Ex + F}{x^2 + 1}$
- E. $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{(x - 1)^3} + \frac{Dx + E}{x^3 + 1}$

Note: The letters A, B, C, D, E, F, G, H in the partial fractions above represent some appropriate constants.

14. (8 points) Evaluate the improper integral $\int_0^2 x \ln(x) dx$.

- A. 0
- B. $\ln(2)$
- C. $2\ln(2)$
- D. $2\ln(2) - 1$
- E. Does not converge.

15. (8 points) $\int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^3 \theta d\theta =$

- A. $\frac{1}{2}$
- B. $\frac{1}{4}$
- C. $\frac{1}{6}$
- D. $\frac{2}{15}$
- E. $\frac{1}{15}$

16. (8 points) Compute the integral $\int_0^{\pi/3} \tan^3 x \sec^3(x) dx$

A. $\frac{58}{15}$

B. $\frac{7}{3}$

C. $\frac{12}{5}$

D. $\frac{18}{23}$

E. $\frac{43}{9}$

17. (8 points) For the integral $I = \int \frac{1}{x^4\sqrt{x^2-4}}dx$, choose the right trigonometric substitution and find the resulting integral.

A. $x = 2 \tan \theta$, $I = \frac{1}{16} \int \frac{\tan \theta}{\sec^3 \theta} d\theta$

B. $x = 4 \tan \theta$, $I = \frac{1}{8} \int \frac{\tan \theta}{\sec^2 \theta} d\theta$

C. $x = 2 \sec \theta$, $I = \frac{1}{16} \int \cos^3 \theta d\theta$

D. $x = 2 \sec \theta$, $I = \frac{1}{16} \int \cos^4 \theta \tan \theta d\theta$

E. $x = 4 \sin \theta$, $I = \frac{1}{2} \int \sin^4 \theta d\theta$

18. (8 points) Evaluate the following integral

$$\int \frac{dt}{\sqrt{t^2 - 6t + 13}}.$$

A. $\ln \left| \frac{\sqrt{(t-3)^2 + 4}}{2} + \frac{t-3}{2} \right| + C$

B. $\ln \left| \sqrt{(t-3)^2 + 4} + \frac{t-3}{2} \right| + C$

C. $\ln \left| \frac{\sqrt{(t-3)^2 + 4}}{2} - \frac{t-3}{2} \right| + C$

D. $\ln \left| \frac{\sqrt{(t-3)}}{2} + \frac{t-3}{2} \right| + C$

E. $\ln \left| \frac{\sqrt{(t-3)^2}}{2} + \frac{t-3}{2} \right| + C$

19. (8 points) Which of the following series converge absolutely ?

I. $\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$

II. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k^{\frac{5}{4}} + k}$

III. $\sum_{k=2}^{\infty} \frac{n}{2^n}$

- A. I and II only.
- B. I and III only.
- C. II and III only.
- D. Only III.
- E. None of them.

20. (8 points) The interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{3^n(x-1)^n}{n}$ is

A. $(2, 4)$

B. $[2, 4]$

C. $[\frac{2}{3}, \frac{4}{3}]$

D. $[\frac{2}{3}, \frac{4}{3})$

E. $(\frac{2}{3}, \frac{4}{3})$

21. (8 Points) Compute the value of the series

$$\sum_{k=0}^{\infty} \frac{k+1}{5^k}$$

HINT: Start from the power series of $f(x) = \frac{1}{1-x}$. What is $f'(x)$, and how does it compare to the series given above?

- A. 3/4
- B. 25/16
- C. 24/5
- D. 2
- E. 5/4

22. (8 points) Choose the right formulas to compute the volume of the solid obtained from rotating **about the line** $x = 2$ the region bounded by $y = e^x$, $y = 2$, and $x = 0$.

(i) Washer method,

(ii) Shell method.

A. (i) $\int_1^2 \pi(4 - (\ln(y))^2) dy$, (ii) $\int_0^{\ln(2)} 2\pi(2 - e^x) dx$

B. (i) $\int_1^2 \pi(4 - (\ln(y))^2) dy$, (ii) $\int_0^{\ln(2)} 2\pi(2 - x)(2 - e^x) dx$

C. (i) $\int_0^{\ln(2)} \pi(2 - e^x)^2 dx$, (ii) $\int_1^2 2\pi(2 - (\ln(y))^2) dy$

D. (i) $\int_0^{\ln(2)} \pi(2 - e^x) dx$, (ii) $\int_1^2 2\pi y(2 - \ln(y)) dy$

E. (i) $\int_1^2 \pi(2 - (\ln(y))^2) dy$, (ii) $\int_0^{\ln(2)} 2\pi(2 - e^x) dx$

23. (8 points) The Alternating Series Test shows that

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{5k}}$$

converges to a value S .

Set

$$S_N = \sum_{k=1}^N (-1)^k \frac{1}{\sqrt{5k}}.$$

Find the smallest N , using The Estimation Theorem for Alternating Series, such that we can conclude

$$|S - S_N| < \frac{1}{10^2}.$$

- A. 200
- B. 201
- C. 2000
- D. 2002
- E. 1998

24. (8 points) In the Taylor series of $f(x) = \frac{1}{x}$ centered at $a = 2$, the coefficient of $(x - 2)^4$ is

A. $\frac{1}{4!}$

B. $\frac{1}{2^5}$

C. $\frac{2^4}{4!}$

D. $\frac{1}{2^4(4!)}$

E. 2.

25. (8 points) Which of the following series is (are) **CONDITIONALLY** convergent?

(I) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{2}\right)^n,$

(II) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{\sqrt{n}}\right),$

(III) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{\ln n}\right)$

- A. (I) only
- B. (I) and (II) only
- C. (II) and (III) only
- D. All
- E. None