MA 26100
Final Exam
12/10/2023
TEST/QUIZ NUMBER:
11

NAME ______________________ YOUR TA'S NAME ______________________

STUDENT ID # ____________________ REC Section # ____________________

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough spaces for your name, fill in as much as you can.
2. Section number with a leading zero, e.g. **0XYZ**. (If you don't know your section number, ask your TA.)
3. Test/Quiz number: **11**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros**
5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–20.

There are **20** questions, each worth 5 points, for a total of 100 points. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

You may not leave the room before 8:20am. If you finish the exam between 8:20am and 9:50am, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 9:50am, you must remain seated until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ______________________________________

1. Which of the following integrals is an expression for the volume of the region given by the intersection of the sphere of radius 2 centered at the origin and the sphere of radius 2 centered at $(0,0,2)$?

A. $\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{2}\rho^2\sin(\phi)d\rho d\phi d\theta+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^{2\cos(\phi)}\rho^2\sin(\phi)d\rho d\phi d\theta$

B. $\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{3}\rho^2\sin(\phi)d\rho d\phi d\theta+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^{6\cos(\phi)}\rho^2\sin(\phi)d\rho d\phi d\theta$

C. $\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{2}\rho\sin(\phi)d\rho d\phi d\theta+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^{4\cos(\phi)}\rho\sin(\phi)d\rho d\phi d\theta$

D. $\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{2}\rho^2\sin(\phi)d\rho d\phi d\theta+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{\pi}\int_0^{4\cos(\phi)}\rho^2\sin(\phi)d\rho d\phi d\theta$

E. $\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{2}\rho^2\sin(\phi)d\rho d\phi d\theta+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^{4\cos(\phi)}\rho^2\sin(\phi)d\rho d\phi d\theta$

2. Suppose $\vec{\mathbf{F}}(x, y, z)$ is a vector field and $g(x, y, z)$ is a real–valued function. Exactly one of the expressions below is meaningless. Which one?

A. $\nabla \times \left(\nabla \cdot \vec{\mathbf{F}}\right)$

B. $\nabla \cdot \left(g\vec{\mathbf{F}}\right)$

C. $\nabla \times (\nabla g)$

D. $\nabla \times \left(\nabla \times \vec{\mathbf{F}}\right)$

E. $\nabla \cdot \left(\nabla \times \vec{\mathbf{F}}\right)$

3. Which of the following is an equation for the tangent plane to the cone $z^2 = x^2 + y^2$ at the point $(3, -4, -5)$?

A. $3x - 4y + 5z = 0$

B. $3x - 4y + 5z = 50$

C. $3x - 4y - 5z = 50$

D. $2x + 2y - 2z = 0$

E. $3x - 4y - 5z = 0$

4. For what value of c is the function

$$f(x,y) = \begin{cases} \dfrac{1+2xy-\cos(xy)}{xy} & xy \neq 0 \\ c & xy = 0 \end{cases}$$

continuous?

A. Such a c does not exist.

B. 1

C. -1

D. 2

E. 0

5. Find the length of the curve $\vec{r}(t) = \left\langle \frac{t^2}{2}, (2t)^{3/2}, 9t + 1 \right\rangle$ on the interval $1 \leq t \leq 2$.

A. $\frac{21}{2}$

B. $\frac{19}{2}$

C. 20

D. 12

E. $\frac{3}{2}$

6. Calculate the flux of $\vec{\mathbf{F}}(x, y, z) = e^{-y}\vec{\mathbf{i}} - 2z\vec{\mathbf{j}} + xy\vec{\mathbf{k}}$, across the curved side of the surface

$$S = \{(x, y, z) : z = \cos(y), 0 \leq x \leq 4, 0 \leq y \leq \pi\}$$

with upward pointing normal.

A. $4\pi^2$

B. $2\pi^2$

C. $4\pi^2 + 8$

D. -16

E. $4\pi^2 - 16$

7. Find the maximum value of the function $f(x, y) = 3 + x^2 + 4y^2$ subject to the constraint $x^2 + 2xy + 4y^2 = 12$.

A. 27

B. 11

C. 54

D. 8

E. 24

8. The equation $xyz + x + y - z = 0$ implicitly defines $z = z(x, y)$. Use implicit differentiation to find $\frac{\partial z}{\partial y}$ at $(x, y) = (3, 1)$.

A. $\frac{5}{2}$

B. $-\frac{2}{5}$

C. $-\frac{1}{2}$

D. $-\frac{3}{5}$

E. $\frac{1}{5}$

9. Let $f(x,y) = x^3 - 3xy + y^2$. Find and characterize the critical points of $f(x,y)$.

A. $(0,0)$ and $(\frac{3}{2}, \frac{9}{4})$ are both saddle points

B. $(0,0)$ is a saddle point and $(\frac{3}{2}, \frac{9}{4})$ is a local maximum

C. $f(x,y)$ has no critical points

D. $(0,0)$ is a saddle point and $\left(\frac{3}{2}, \frac{9}{4}\right)$ is a local minimum

E. $(0,0)$ and $(\frac{3}{2}, \frac{9}{4})$ are both local minima

10. Find the equation of the line of intersection of the planes $2x - y + 4z = 8$ and $x - 3z = 1$.

A. $\langle x, y, z\rangle = \langle -12 + t, 4 - 2t, -5 + t\rangle$

B. $\langle x, y, z\rangle = \langle 3 + t, 10 - 6t, 1\rangle$

C. $\langle x, y, z\rangle = \langle 1 - 12t, -2 + 4t, 1 - 5t\rangle$

D. $\langle x, y, z\rangle = \langle 1 + 3t, -6 + 10t, t\rangle$

E. $\langle x, y, z\rangle = \langle 1 + 3t, -2 + 10t, 1 + t\rangle$

11. Evaluate $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, dS$ where $\vec{\mathbf{F}} = \langle xy, -\frac{1}{2}y^2, z\rangle$ and the surface S consists of three pieces:

$$\begin{cases} z = 4 - 3x^2 - 3y^2 & 1 \le z \le 4 \text{ on the top} \\ x^2 + y^2 = 1 & 0 \le z \le 1 \text{ on the sides} \\ z = 0 & \text{on the bottom} \end{cases}$$

A. 2π

B. $\dfrac{3\pi}{2}$

C. 0

D. π

E. $\dfrac{5\pi}{2}$

12. The following integral gives us the volume of which of the following solids:

$$\int_{\pi}^{2\pi}\int_{0}^{4}\sqrt{16-r^2}\,r\,dr\,d\theta$$

A. A sphere centered at the origin with radius 4.

B. The intersection of a sphere centered at the origin with radius 4 and the regions $\{y \leq 0\}$ and $\{z \geq 0\}$

C. A cylinder with base centered at the origin of height 1 and base radius 4.

D. The intersection of a sphere centered at the origin with radius 4 and the region $\{y \leq 0\}$.

E. The intersection of a sphere centered at the origin with radius 4 and the region $\{y \geq 0\}$.

13. Let $\vec{\mathbf{F}}(x,y,z) = \cos\left(y^2z^3\right)\vec{\mathbf{i}} - xz^8\vec{\mathbf{j}} + z^{16}\vec{\mathbf{k}}$, and let S be the semi-ellipsoid $\dfrac{z}{5} = \sqrt{1 - \dfrac{x^2}{4} - \dfrac{y^2}{9}}$ with upward unit normal. Evaluate $\displaystyle\iint_S \left(\nabla \times \vec{\mathbf{F}}\right) \cdot \vec{\mathbf{n}}\, dS$.

A. 1
B. 3
C. 0
D. 4
E. 2

14. Consider the circle C centered at the origin with radius 3. A particle travels once around C, counterclockwise. It is subject to the force

$$\vec{\mathbf{F}}(x,y) = \left\langle y^3,\, x^3 + 3xy^2 + 1 \right\rangle .$$

Use Green's theorem to find the work done by $\vec{\mathbf{F}}$.

A. $\dfrac{4\,\pi}{3}$

B. $\dfrac{23\,\pi}{3}$

C. $\dfrac{3\,\pi}{4}$

D. $\dfrac{243\,\pi}{4}$

E. $\dfrac{117\,\pi}{4}$

15. Find the volume enclosed between the graphs of the functions $f(x,y) = x^2 + y^2 - 1$ and $g(x,y) = 1 - x^2 - y^2$.

A. 2π

B. $\frac{\pi}{2}$

C. π

D. 0

E. $\frac{8\pi}{3}$

16. Let $\vec{\mathbf{F}}(x, y) = \langle y^3+1,\, 3xy^2+1\rangle$. Consider a straight line path C from $(0, 1)$ to $(2, 1)$. Evaluate $\displaystyle\int_C \vec{\mathbf{F}} \cdot d\mathbf{r}$.

A. 18

B. 19

C. 11

D. 16

E. 21

17. Given the force field $\vec{\mathbf{F}}(x, y) = \langle 1 - y, x \rangle$, find the work required to move an object along the ellipse $\vec{\mathbf{r}}(t) = \langle \cos(t), 4\sin(t) \rangle$ from $(0, 4)$ to $(-1, 0)$.

A. $2\pi - 1$

B. 4π

C. $2\pi - 2$

D. $4\pi - 1$

E. 2π

18. Let S be that part of the surface $z = \dfrac{y^2}{4}$ which lies over the triangle with vertices $(0, 0)$, $(0, 1)$, $(1, 1)$ in the xy-plane. Which of the following is the surface area of S?

A. $\dfrac{1}{2}$

B. $\dfrac{5^{3/2} - 8}{6}$

C. $\dfrac{5}{2}$

D. $\dfrac{5^{3/2} - 1}{6}$

E. 1

19. Identify the surface $z^2 + y^2 + 4 = 5x^2 + 4z$

A. Hyperbolic Paraboloid

B. Hyperboloid of 2 sheets

C. Elliptic Paraboloid

D. Cone

E. Hyperboloid of 1 sheet

20. Find the direction $\vec{\mathbf{u}}$ of zero change of the function $f(x, y) = e^{x^2-4y^2}$ at the point $(x, y) = (-1, 2)$.

A. $\vec{\mathbf{u}} = \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

B. $\vec{\mathbf{u}} = \left\langle \frac{4}{\sqrt{17}}, \frac{-1}{\sqrt{17}} \right\rangle$

C. $\vec{\mathbf{u}} = \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$

D. $\vec{\mathbf{u}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

E. $\vec{\mathbf{u}} = \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$