MA 26100
FINAL EXAM INSTRUCTIONS
VERSION 01
December 9, 2024

Your name ______________ Your TA's name ______________

Student ID # ______________ Section # and recitation time ______________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces. Put TWO Zeroes in front of your ID number on your card.

5. Fill in the four-digit **SECTION NUMBER**. Put Zero in front of your section number.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the space provided for each of the questions. While mark all your work on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

8. There are 20 questions, each worth 10 points.The maximum possible score is 200 points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

11. If you finish the exam before 9:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 9:55, REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

1. Reverse the order of integration and evaluate

$$\int_0^1 \int_{\sqrt[3]{x}}^1 \frac{1}{y^4+1}\, dy\, dx$$

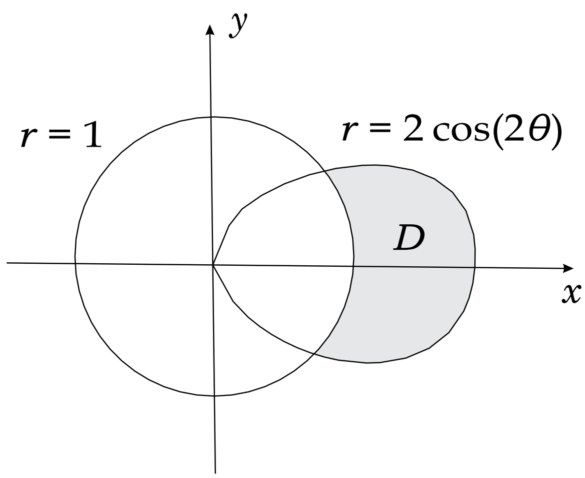
A. $\ln(2)/2$

B. $\ln(2)/4$

C. $\ln(2)$

D. $2\ln(2)$

E. $4\ln(2)$



2. Which integral represents the area of a domain D between the curves $r = 1$ and $r = 2\cos(2\theta)$ shown in the figure above?

A. $\displaystyle\int_{-\pi/4}^{\pi/4}\int_{1}^{2\cos(2\theta)} r\,dr\,d\theta$

B. $\displaystyle\int_{-\pi/3}^{\pi/3}\int_{1}^{2\cos(2\theta)} r\,dr\,d\theta$

C. $\displaystyle\int_{-\pi/6}^{\pi/6}\int_{1}^{2\cos(2\theta)} r\,dr\,d\theta$

D. $\displaystyle\int_{-\pi/4}^{\pi/4}\int_{0}^{2\cos(2\theta)} r\,dr\,d\theta$

E. $\displaystyle\int_{-\pi/6}^{\pi/6}\int_{0}^{2\cos(2\theta)} r\,dr\,d\theta$

3. A thin plate bounded by $y = 0$, $y = x^2$ and $x = 1$ has density of mass $\rho = x$. Its mass is $1/4$. The x-coordinate of its center of mass is equal

A. $1/5$

B. $3/10$

C. $3/5$

D. $4/5$

E. $9/10$

4. Find the values for a and b that convert the triple integral from rectangular coordinates to spherical coordinates.

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} y\, dz\, dy\, dx = \int_0^{\pi/2} \int_a^{\pi/2} \int_0^{3\csc\phi} b\, d\rho\, d\phi\, d\theta$$

A. $a = 0,\ b = \rho^2 \sin\phi$

B. $a = \pi/4,\ b = \rho^3 \sin\phi \sin\theta$

C. $a = \pi/4,\ b = \rho^3 \sin^2\phi \sin\theta$

D. $a = \pi/3,\ b = \rho^3 \sin^2\phi \sin\theta$

E. $a = -\pi/2,\ b = \rho^3 \sin^2\phi$

5. Evaluate the flux integral

$$\int\!\!\int_{\Sigma} \vec{F} \cdot \vec{n}\; dS$$

where Σ is the top half of the sphere of radius 1 centered at the origin, $\vec{F} = \langle y, -x, 2\rangle$. Hint: Do not use spherical coordinates.

A. 0

B. $\pi/2$

C. π

D. $3\pi/2$

E. 2π

6. The tangent plane to $z = (x^2 + y^2)^3$ at $(x, y) = (1, -2)$ is:

A. $75x + 75y - z + 200 = 0$

B. $75x + 75y + z - 50 = 0$

C. $150x - 300y + z - 875 = 0$

D. $150x + 300y - z + 675 = 0$

E. $150x - 300y - z - 625 = 0$

7. Given that $\vec{F}(x, y) = \langle 2xe^{2y} - 2,\ 2x^2e^{2y} + 2y \rangle = \vec{\nabla} f(x, y)$, compute $f(1, 1) - f(1, 0)$.

A. $2e^2$

B. e^2

C. $e^2 + 2$

D. $e^2 + 3$

E. $e^2 - 1$

8. Compute $\displaystyle\iint_{\Sigma} (\nabla \times \vec{F}) \cdot \vec{n}\, dS$ for $\vec{F} = \langle 2x{+}z, 2y{+}x, 2z{+}y \rangle$ and Σ being the portion of the surface $z = \sqrt{3 + x^2 + y^2}$ within the cylinder $x^2 + y^2 = 1$. The surface is oriented upwards.

A. 2π

B. 0

C. -2π

D. π

E. 1

9. Compute the volume enclosed between the surfaces $z = x^2 + 3y^2 - 20$ and $z = -3x^2 - y^2 + 16$.

A. 2π

B. 144π

C. 162π

D. 81π

E. 0

10. Compute the curvature of the spiral $\vec{r}(t) = \langle 3\cos(t), 3\sin(t), 2t \rangle$.

A. $\frac{3}{13}$

B. $\frac{1}{2}$

C. $\frac{2}{13}$

D. $\frac{4}{13}$

E. $\frac{1}{3}$

11. Suppose $f(x, y)$ is such that $\frac{\partial f}{\partial x}(5, -3) = 1$ and $\frac{\partial f}{\partial y}(5, -3) = 2$. For $x(u, v) = 3u + 2v$ and $y(u, v) = u - 4v$ compute $\frac{\partial f}{\partial u}(1, 1)$.

A. -4

B. 2

C. -3

D. 0

E. 5

12. What is the intersection of the planes $x + 2y + 3z = 1$ and $3x + y - z = 2$?

A. The line $\begin{pmatrix} 1/5 \\ 1 \\ -4/5 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

B. The line $\begin{pmatrix} 3/5 \\ 1/5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

C. The line $\begin{pmatrix} 2/5 \\ 2/5 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

D. The planes do not intersect

E. The planes are identical

13. Compute the flux integral $\iint_\Sigma \vec{F} \cdot \vec{n}\, dS$,

where $\vec{F}(x,y,z) = e^{2x}\sin(2y)\vec{\imath} + e^{2x}\cos(2y)\vec{\jmath} + y^2z^2\vec{k}$, Σ is the boundary surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 2$, and $\vec{n}$ is the outward unit normal.

A. 8/3

B. 2/3

C. 7/3

D. 4/3

E. 10/3

14. For

$$-x^2 + 6x + y^2 - 10y - z^2 + 8z = 1$$

determine what type of quadric surface this equation represents.

A. An elliptic paraboloid

B. A hyperboloid of one sheet

C. A hyperbolic paraboloid

D. A hyperboloid of two sheets

E. An ellipsoid

15. Compute the integral $\int_C xdy - ydx$

where C is the curve $(x-1/2)^2+(y-1/3)^2 = 3$ oriented counterclockwise.

A. 6π

B. $8\pi/3$

C. π

D. $2\pi/3$

E. 4π

16. Compute the area of the part of the plane $x + 2y + z = 6$ that is directly above the region $\{0 \leq x \leq 2, 0 \leq y \leq 1\}$?

A. $2\sqrt{6}$

B. 9

C. 2

D. $9\sqrt{6}$

E. $\sqrt{11}$

17. Let $f(x, y) = x^2y + xy^2$. Find all points $(x, y) \neq (0, 0)$ such that the vector $\langle 1, 1\rangle$ points in the same direction as the direction of greatest increase of the function $f(x, y)$ at (x, y).

A. $(x, y) = (x, -x)$ for $x \neq 0$

B. $(x, y) = (x, x)$ for $x \neq 0$

C. $(x, y) = (x, x)$ for $x \neq 0$ and $(x, y) = (x, -x)$ for $x \neq 0$

D. $(x, y) = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

E. There are no points $(x, y) \neq (0, 0)$ for which this occurs

18. Compute $\iint_R e^{-x^2-y^2}\,dA$, where R is the region surrounded by the circle $x^2+y^2=16$.

A. $\pi(e^{16}-1)$

B. $\pi(e^{-16}-1)$

C. $\pi(1-e^{16})$

D. $\pi(1-e^{-16})$

E. 0

19. Compute the length of the curve parametrized by $\vec{r}(t) = \langle 2\cos(t), \sin(t), \sqrt{3}\sin(t)\rangle$, $\frac{\pi}{4} \le t \le \frac{\pi}{2}$.

A. 0

B. 4π

C. 1

D. π

E. $\frac{\pi}{2}$

20. Find the critical points of $f(x,y) = x^2 - y^3x$.

A. $(0,0)$

B. $(1,0)$

C. $(0,1)$

D. $(-1,1)$

E. $(1,-1)$