MA 26100
Final Exam - Spring 2024
05/02/2024
TEST/QUIZ NUMBER:
31

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # ____________________ RECITATION # ____________________

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough spaces for your name, fill in as much as you can.
2. Your recitation section number. (If you don't know your recitation section number, ask your TA.)
3. Test/Quiz number: **31**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros**
5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–20.

There are **20** questions, each worth 5 points, for a total of 100 points. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50pm, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 7:20pm. If you don't finish before 8:50pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, phone, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ____________________

1. Find a function $\vec{r}(t)$ that describes the curve where the surfaces $x^2 + y^2 = 16$ and $z = 2x + 3y$ intersect.

A. $\vec{r}(t) = \langle 4\cos t, 4\sin t, 2\cos t + 3\sin t\rangle$

B. $\vec{r}(t) = \langle 16\cos t, 16\sin t, 32\cos t + 48\sin t\rangle$

C. $\vec{r}(t) = \langle 4\cos t, 4\sin t, 8\cos t + 12\sin t\rangle$

D. $\vec{r}(t) = \langle 16\cos t, 16\sin t, 2\cos t + 3\sin t\rangle$

E. $\vec{r}(t) = \langle 2\cos t, 2\sin t, 4\cos t + 6\sin t\rangle$

2. Find the unit tangent vector to $\vec{r}(t) = \langle te^t, t^3, 2\sqrt{t}\rangle$ at $t = 1$.

A. $\dfrac{1}{\sqrt{4e^2+4}}\langle 2e, 3, 1\rangle$

B. $\dfrac{1}{\sqrt{4e^2+10}}\langle 2e, 3, 1\rangle$

C. $\dfrac{1}{\sqrt{e^2+4}}\langle e, 3, 1\rangle$

D. $\dfrac{1}{\sqrt{e^2+10}}\langle e, 3, 1\rangle$

E. $\dfrac{1}{\sqrt{e^4+10}}\langle e^2, 3, 1\rangle$

3. Find the line of intersection for the planes $x - y + 2z = 1$ and $2x - z = 1$.

A. $\langle \frac{1}{2}, \frac{1}{2}, 0\rangle + t\langle 1, 5, 2\rangle$

B. $\langle \frac{1}{2}, -\frac{1}{2}, 0\rangle + t\langle 1, -5, 2\rangle$

C. $\langle \frac{1}{2}, -\frac{1}{2}, 0\rangle + t\langle 1, -3, 2\rangle$

D. $\langle \frac{1}{2}, -\frac{1}{2}, 0\rangle + t\langle 1, 5, 2\rangle$

E. $\langle \frac{1}{2}, \frac{1}{2}, 0\rangle + t\langle 1, -5, 2\rangle$

4. For the level surface $6x^2z + yz^2 = 7$, use implicit differentiation to find $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}$ at $(1, 1, 1)$

A. $\dfrac{-26}{3}$

B. $\dfrac{-13}{8}$

C. $\dfrac{26}{3}$

D. $\dfrac{13}{8}$

E. 7

5. A baseball is thrown horizontally from the top of a building that is 19.6 meters tall. The initial velocity of the baseball is 15 meter/sec . Assuming no air resistance, how far from the base of the building does the baseball land? (Recall that the gravitational acceleration is 9.8 meter/sec^2).

A. 15 meters

B. 45 meters

C. 30 meters

D. 60 meters

E. 75 meters

6. Suppose $f(x,y) = e^{4y}\sin(-4x)$, $g(x,y) = e^{4y}\cos(-4x)$. Is $\vec{F} = \langle f(x,y), g(x,y)\rangle$ conservative on $\mathbb{R}^2$?

A. No, because $\dfrac{\partial f}{\partial y}$ does not equal $\dfrac{\partial g}{\partial x}$

B. Yes, because $\dfrac{\partial f}{\partial x}$ equals $\dfrac{\partial g}{\partial y}$

C. No, because $\dfrac{\partial f}{\partial y}$ equals $\dfrac{\partial g}{\partial x}$

D. Yes, because $\dfrac{\partial f}{\partial y}$ does not equal $\dfrac{\partial g}{\partial x}$

E. Yes, because $\dfrac{\partial f}{\partial y}$ equals $\dfrac{\partial g}{\partial x}$

7. The extreme values of $f(x, y, z) = 2x + 4y + 4z$ with constraint $x^2 + y^2 + z^2 = 9$ are

A. The maximum if f is 18 and the minumun is -18

B. The maximum if f is 6 and the minumun is -18

C. The maximum if f is 12 and the minumun is -12

D. The maximum if f is 18 and the minumun is -6

E. The maximum if f is 6 and the minumun is -6

8. Evaluate the integral $\iint\limits_S curl\vec{F} \cdot d\vec{S}$ using Stoke's Theorem, where $\vec{F} = \langle -y, x, z^3 + 5 \rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies above the xy-plane, oriented upward.

A. -18π

B. 18π

C. -9π

D. 0

E. 9π

9. Let $f(x, y) = x^2ye^{xy}$, then the direction of steepest descent at $(1, -1)$ is in the direction of the vector

A. $\langle 1, 0\rangle$

B. $\langle 1, -1\rangle$

C. $\langle -1, 0\rangle$

D. $\langle 1, 1\rangle$

E. $\langle -1, 1\rangle$

10. Let $f(x, y) = (x^2 - y^2)e^x$. The function has

A. a local maximum and a saddle point

B. two local maximums

C. a local minimum and a saddle point

D. a local maximum and a local minimum

E. two local minimums

11. Evaluate the following triple integral with the help of cylindrical coordinates:

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \int_{0}^{3} \frac{1}{1+x^2+y^2}\, dz\, dy\, dx.$$

A. $\frac{3}{2}\pi \ln 5$

B. $\frac{3}{8}\pi \ln 5$

C. $3\pi \ln 5$

D. $6\pi \ln 5$

E. $\frac{3}{4}\pi \ln 5$

12. Evaluate $\iiint\limits_D \dfrac{1}{\sqrt{x^2+y^2+z^2}}\, dV$. Where D is the region outside of sphere of radius 1 and inside a sphere of radius 2, both centered at the origin, and $z \geq 0$.

A. 12π

B. 0

C. 9π

D. 6π

E. 3π

13. The surface area of the parametric surface $\vec{r}(u, v) = \langle u^2, uv, \frac{v^2}{2} \rangle$ with $0 \leq u \leq 1,\ 0 \leq v \leq 3$.

A. 19

B. $\frac{29}{3}$

C. $\frac{17}{3}$

D. 11

E. 15

14. Evaluate $\int_0^{\frac{1}{2}} \int_{\sqrt{3}x}^{\sqrt{1-x^2}} 5(x^2+y^2)^{\frac{3}{2}}\ dy\ dx$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{2}$

15. Consider the limits

$$A = \lim_{(x,y)\to(0,0)} \frac{4x - 3y}{\sqrt{x^2 + y^2}} \quad \text{and} \quad B = \lim_{(x,y)\to(0,0)} \frac{e^{x-y}}{2 - e^{x+y}}.$$

Which one of the following statements are true?

A. $A = 1,\ B = 1$

B. $A = 1,\ B = \frac{1}{2}$

C. A does not exist, $B = 1$

D. Both A and B do not exist

E. A does not exist, $B = \frac{1}{2}$

16. If $f(x, y, z) = \tan^{-1}(xyz)$, compute f_{xx}.

A. $\dfrac{2x^2y^2z^2}{(1 + (xyz)^2)^2}$

B. $\dfrac{(1 - 3x^2)}{(1 + (xyz)^2)^2}$

C. $\dfrac{3y^2z^2(1 - x^2)}{(1 + (xyz)^2)^2}$

D. $\dfrac{-2x(yz)^3}{(1 + (xyz)^2)^2}$

E. $\dfrac{2y^2z^2}{(1 + (xyz)^2)^2}$

17. Compute $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = \langle -y, x \rangle$, where C is the curve bounding the triangle with coordinates $(0,0), (2,0),$ and $(1,2)$ oriented counterclockwise.

A. 0

B. -4

C. 2

D. -2

E. 4

18. The arclength of the curve $\vec{r}(t) = \langle e^t, \sqrt{2}t, e^{-t} \rangle$ for $0 \leq t \leq \ln 2$ is:

A. $e^2 - e^{-2}$

B. $\dfrac{5}{2}$

C. $\dfrac{1}{2}$

D. $e^2 + e^{-2}$

E. $\dfrac{3}{2}$

19. If S is the part of $x = y^2 + z^2$ with $x \leq 3$ and $\vec{n}$ is the unit normal pointing towards **positive x-axis**, and $\vec{F} = \langle 4x, y, z \rangle$, then $\iint\limits_S \vec{F} \cdot \vec{n}\ dS = ?$

A. 0

B. 3π

C. -3π

D. -9π

E. 9π

20. Let $\phi = xe^{xyz}$ and let $D = \{(x, y, z)|0 \le x, y, z \le 1\}$ with surface S. Compute the integral $\iint_S (\vec{\nabla} \times \vec{\nabla}\phi) \cdot \vec{n}\, dS$.

A. 0

B. e^2

C. e

D. $\frac{1}{e}$

E. $\frac{1}{e^2}$

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