MA 265, Spring 2024, Midterm I (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**

2. After you have finished the exam, hand in your test booklet to your instructor.

172	MWF	9:30AM	Eric Griffin Samperton	264	MWF	1:30PM	Yiran Wang
173	MWF	12:30PM	Andrey Glubokov	265	MWF	12:30PM	Yiran Wang
196	TR	3:00PM	Jing Wang	276	MWF	8:30AM	Iryna Egorova
201	MWF	9:30AM	Daniel Tuan-Dan Le	277	MWF	9:30AM	Iryna Egorova
202	TR	3:00PM	Ning Wei	281	TR	7:30AM	Arun Albert Debray
213	TR	4:30PM	Ning Wei	282	TR	12:00PM	Arun Albert Debray
214	MWF	12:30PM	Ping Xu	283	MWF	2:30PM	Oleksandr Tsymbaliuk
225	MWF	2:30PM	Ping Xu	284	TR	1:30PM	Jing Wang
226	MWF	1:30PM	Ping Xu	285	MWF	12:30PM	Yi Wang
237	MWF	10:30AM	Sai Kee Yeung	287	MWF	1:30PM	Yi Wang
238	MWF	3:30PM	Siamak Yassemi	288	MWF	7:30AM	Krishnendu Khan
240	MWF	12:30PM	Daniel Lentine Johnstone	289	MWF	8:30AM	Krishnendu Khan
241	MWF	11:30AM	Daniel Lentine Johnstone	298	MWF	4:30PM	Siamak Yassemi
252	TR	10:30AM	Raechel Polak	299	MWF	12:30PM	Ying Zhang
253	TR	9:00AM	Raechel Polak	300	MWF	1:30PM	Ying Zhang

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.

4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.

5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

1. (10 points) For what values of r and s is the linear system INCONSISTENT?

$$\begin{aligned} x + y + z &= 2 \\ x \phantom{{}+y} + 2z &= 3 + s \\ x - y + rz &= 2 \end{aligned}$$

A. $r = 4$ and $s \neq -1$

B. $r = -4$ and $s \neq 1$

C. $r = 3$ and $s \neq -1$

D. $r = -3$ and $s \neq 1$

E. None of the above

2. (10 points) If the determinant of the matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ is 2, calculate the determinant of $\begin{bmatrix} g+4a & h+4b & i+4c \\ 2a & 2b & 2c \\ 3d & 3e & 3f \end{bmatrix}$.

A. -48

B. 48

C. -12

D. 12

E. 0

3. (10 points) Let $A = \begin{bmatrix} -2 & 6 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ and $C = AB^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find a.

A. 7

B. 5

C. 9

D. -14

E. -6

4. (10 points) Suppose A, B and C are $n \times n$ matrices, which of the following statements must be true?

(i) If B is invertible, then $AB = AC$ implies $B = C$.

(ii) If $A\mathbf{x} = \mathbf{b}$ has a solution for all the vectors $\mathbf{b}$ in $\mathbb{R}^n$, then A is invertible.

(iii) If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $A\mathbf{x} = \mathbf{b}$ has exactly one solution for any vector $\mathbf{b}$ in $\mathbb{R}^n$.

(iv) If $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for any vector $\mathbf{b}$ in $\mathbb{R}^n$.

(v) If both A and B are invertible, then $A + B$ is also invertible.

A. (i), (iii) and (iv) only.

B. (ii) and (iii) only.

C. (ii), (iii) and (iv) only.

D. (ii), (iii), (iv) and (v) only.

E. (i), (ii), (iii), (iv) and (v).

5. (10 points) Let T be a linear transformation and $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & h \\ 2 & 5 & h^2 \end{bmatrix}$ be the standard matrix of T. Determine the value(s) of h such that T is one-to-one.

A. $h \neq -2$ and $h \neq 3$

B. $h \neq -1$ and $h \neq 4$

C. $h \neq -1$ and $h \neq 5$

D. $h \neq 2$ and $h \neq -5$

E. $h \neq 2$ and $h \neq 3$

6. (10 points) Which of the following statements is always FALSE?

A. If A is an invertible $n \times n$ matrix, then A^2 is row equivalent to the $n \times n$ identity matrix.

B. If A and B are invertible $n \times n$ matrices, then the columns of AB span $\mathbb{R}^n$.

C. If A is a non-invertible square matrix, then at least one column of A is in the span of the remaining columns.

D. If A is a 3×5 matrix and the dimension of the nullspace of A is 2, then for each $\mathbf{b}$ in $\mathbb{R}^3$, $A\mathbf{x} = \mathbf{b}$ has at least one solution.

E. If A is a 5×3 matrix, then $\mathbf{x} \mapsto A\mathbf{x}$ can't be one-to-one.

7. (10 points) Which of the following collection of vectors is linearly independent?

A. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ -3 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 3 \\ -9 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 8 \\ 0 \\ -6 \end{bmatrix} \right\}$

8. Let

$$A = \begin{bmatrix} 1 & 3 & -1 & 1 & 3 \\ 1 & 3 & 0 & 3 & 6 \\ -2 & -6 & 2 & -2 & -6 \end{bmatrix}.$$

(3 points)(1) Find the REDUCED row echelon form for the matrix A.

(2 points)(2) Find the rank of A.

(5 points)(3) Find a basis for the null space of A.

9. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 7 \\ 1 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix}$$

(5 points)(1) Find the determinant of A.

(5 points)(2) Let $B = [b_{ij}]$ be the inverse matrix of A, find b_{23}.

10. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation with

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}0\\2\\4\end{bmatrix}, \quad T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}-1\\2\\9\end{bmatrix}.$$

(4 points)(1) Find $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.

(2 points)(2) Let A be the standard matrix of T, find A.

(4 points)(3) Find h such that the vector $\mathbf{b} = \begin{bmatrix}4\\1\\h\end{bmatrix}$ is in the range of T.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________