MA 265, Spring 2024, Midterm II (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

172	MWF	9:30AM	Eric Griffin Samperton	264	MWF	1:30PM	Yiran Wang
173	MWF	12:30PM	Andrey Glubokov	265	MWF	12:30PM	Yiran Wang
196	TR	3:00PM	Jing Wang	276	MWF	8:30AM	Iryna Egorova
201	MWF	9:30AM	Daniel Tuan-Dan Le	277	MWF	9:30AM	Iryna Egorova
202	TR	3:00PM	Ning Wei	281	TR	7:30AM	Arun Albert Debray
213	TR	4:30PM	Ning Wei	282	TR	12:00PM	Arun Albert Debray
214	MWF	12:30PM	Ping Xu	283	MWF	2:30PM	Oleksandr Tsymbaliuk
225	MWF	2:30PM	Ping Xu	284	TR	1:30PM	Jing Wang
226	MWF	1:30PM	Ping Xu	285	MWF	12:30PM	Yi Wang
237	MWF	10:30AM	Sai Kee Yeung	287	MWF	1:30PM	Yi Wang
238	MWF	3:30PM	Siamak Yassemi	288	MWF	7:30AM	Krishnendu Khan
240	MWF	12:30PM	Daniel Lentine Johnstone	289	MWF	8:30AM	Krishnendu Khan
241	MWF	11:30AM	Daniel Lentine Johnstone	298	MWF	4:30PM	Siamak Yassemi
252	TR	10:30AM	Raechel Polak	299	MWF	12:30PM	Ying Zhang
253	TR	9:00AM	Raechel Polak	300	MWF	1:30PM	Ying Zhang

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. You cannot turn in your exam during the first 20 minutes or the last 10 minutes of the exam period.
5. When time is called, all students must put down their writing instruments immediately.
6. Any violation of these rules (3-5) and any act of academic dishonesty may result in severe penalties. Additionally, all cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ____________________________

STUDENT SIGNATURE ____________________________

STUDENT PUID ____________________________

SECTION NUMBER ____________________________

1. (10 points) Which of the following sets of polynomials is linearly independent?

A. $\{2 + t - t^2, 1 + t, 1 - t^2\}$

B. $\{t^2 + 1, t^2 - 1, t + t^2\}$

C. $\{1 + 2t, 1 + 3t, t\}$

D. $\{1 + t^2, 2 + 2t^2, 1 - t^2\}$

E. $\{1 + t + t^2, 1 + t, t^2\}$

2. (10 points) For which value of a is the vector $\mathbf{v} = \begin{bmatrix} a & 2 & 8 \end{bmatrix}$ contained in the row space of the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 5 \\ 2 & 2 & 14 \end{bmatrix}$?

A. $a = 0$

B. $a = 1$

C. $a = -1$

D. $\mathbf{v}$ is in the row space of A for any a.

E. There is no a for which this is possible.

3. (10 points) For the system of differential equations $\begin{cases} x'(t) = x(t) - 2y(t) \\ y'(t) = -2x(t) + y(t) \end{cases}$, the origin is

A. a repeller

B. an attractor

C. a saddle point

D. a spiral point

E. none of the above

4. (10 points) Select all the correct statements among the following ones:

(i) An $n \times n$ matrix A is diagonalizable if it has n pairwise distinct eigenvalues.

(ii) An $n \times n$ matrix A is not invertible if 0 is an eigenvalue of A.

(iii) If $A\mathbf{v} = \lambda\mathbf{v}$ with $\mathbf{v} \neq \mathbf{0}$, then $\mathbf{v}$ is an eigenvector of A^3.

(iv) Given two $n \times n$ matrices A and B, if A and B are similar, then the matrices A^3 and B^3 are also similar .

(v) If two $n \times n$ matrices A and B have the same characteristic polynomials and A is invertible, then B is also invertible.

A. (i), (ii) and (iii) only

B. (i), (ii) and (iv) only

C. (i), (ii), (iv) and (v) only

D. (i), (ii) and (v) only

E. (i), (ii), (iii), (iv) and (v)

5. (10 points) Let $\mathbb{P}_3$ denote the vector space of all real polynomials in one variable t of degree at most 3 and the zero polynomial. Which of the following statements must be TRUE?

A. The set of all $p(t) \in \mathbb{P}_3$ satisfying $p(0) + p(1) = p(2) + 3$ is a subspace of $\mathbb{P}_3$.

B. The set of all $p(t) \in \mathbb{P}_3$ satisfying $p(0) \cdot p(1) = 0$ is a subspace of $\mathbb{P}_3$ of dimension 2.

C. The set of all $p(t) \in \mathbb{P}_3$ satisfying $p(0) \cdot p(1) = 0$ is a subspace of $\mathbb{P}_3$ of dimension 3.

D. The set of all $p(t) \in \mathbb{P}_3$ satisfying $p(0) = 0$ and $p(1) = 0$ is a subspace of $\mathbb{P}_3$ of dimension 2.

E. The set of all $p(t) \in \mathbb{P}_3$ satisfying $p(0) = 0$ and $p(1) = 0$ is a subspace of $\mathbb{P}_3$ of dimension 3.

6. (10 points) Let A be a 2×2 real matrix which satisfies $A \begin{bmatrix} i \\ 1 \end{bmatrix} = (2+i) \begin{bmatrix} i \\ 1 \end{bmatrix}$. Which of the following is a general solution to the system of differential equations $\mathbf{x}'(t) = A \cdot \mathbf{x}(t)$?

A. $c_1 e^{2t} \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$

B. $c_1 e^{2t} \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

C. $c_1 e^{t} \begin{bmatrix} -\cos 2t \\ \sin 2t \end{bmatrix} + c_2 e^{t} \begin{bmatrix} \sin 2t \\ \cos 2t \end{bmatrix}$

D. $c_1 e^{t} \begin{bmatrix} -\sin 2t \\ \cos 2t \end{bmatrix} + c_2 e^{t} \begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix}$

E. none of the above

7. (10 points) Let $\mathbb{P}_2$ be the vector space consisting of all polynomials of degree at most 2 and the zero polynomial. Consider the linear transformation $T : \mathbb{P}_2 \mapsto \mathbb{P}_2$ defined by

$$T(p(t)) = p(1)t^2 - p(2)t + p(-1).$$

Find the matrix $[T]_{\mathfrak{B}}$ for T relative to the ordered basis $\mathfrak{B} = \{t^2 - 1,\ t,\ 1\}$.

A. $[T]_{\mathfrak{B}} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -2 & -3 \\ 1 & -1 & 0 \end{bmatrix}$

B. $[T]_{\mathfrak{B}} = \begin{bmatrix} 0 & 1 & 1 \\ -3 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

C. $[T]_{\mathfrak{B}} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -2 & -3 \\ 2 & 0 & 0 \end{bmatrix}$

D. $[T]_{\mathfrak{B}} = \begin{bmatrix} 0 & 1 & 1 \\ -3 & -2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$

E. $[T]_{\mathfrak{B}} = \begin{bmatrix} 0 & 1 & 1 \\ -3 & -2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

8. Let $\mathbb{P}_3$ be the vector space consisting of all polynomials of degree at most three and the zero polynomial. Consider the linear transformation:

$$T : \mathbb{P}_3 \to \mathbb{R}^3$$

$$T(p(t)) = \begin{bmatrix} 0 \\ p(0) \\ p(-2) \end{bmatrix}.$$

(2 points) (1) Let $p(t) = t^3 + 2t^2 + 2t + 5$, find $T(p(t))$.

(4 points) (2) Determine a basis for the range of T.

(4 points) (3) Determine a basis for the kernel of T.

9. Let

$$A = \begin{bmatrix} 5 & -3 \\ 10 & -6 \end{bmatrix}.$$

(6 points) (1) Find an invertible matrix P and a diagonal matrix D such that

$$A = PDP^{-1}.$$

(4 points) (2) Find $A^{2024} + 2A^{2025}$.

10. Given that vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2024 \\ -1 \end{bmatrix}$ are the eigenvectors of the matrix

$$A = \begin{bmatrix} 2025 & 2024 \\ -1 & 0 \end{bmatrix}.$$

(2 points) (1) Find their corresponding eigenvalues λ_1 and λ_2.

(2 points) (2) Find a general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 2025 & 2024 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(6 points) (3) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular solution to the initial value problem

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 2025 & 2024 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2025 \\ -2 \end{bmatrix}.$$

Find $x(1) + y(1)$.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ______________________