GREEN - Test Version 01

NAME ______________________ INSTRUCTOR ______________________

1. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Use the back of the test pages or the last two sheets of blank paper for scratch paper. **PLEASE PUT YOUR SCANTRON UNDERNEATH YOUR QUESTION SHEETS WHEN YOU ARE NOT FILLING IN THE SCANTRON SHEET.**

2. Fill in your name and your instructor's name on the question sheets (above).

3. You must use a **#2 pencil** on the mark–sense sheet (answer sheet). Fill in the instructor's name and the course number(**MA265**), fill in the correct TEST/QUIZ NUMBER (**GREEN is 01**), your name, your section number (see below if you are not sure), and your 10-digit PUID(BE SURE TO INCLUDE THE TWO LEADING ZEROS of your PUID.) and shade them in the appropriate spaces. Sign the mark–sense sheet.

172	MWF	9:30AM	Eric Griffin Samperton	264	MWF	1:30PM	Yiran Wang
173	MWF	12:30PM	Andrey Glubokov	265	MWF	12:30PM	Yiran Wang
196	TR	3:00PM	Jing Wang	276	MWF	8:30AM	Iryna Egorova
201	MWF	9:30AM	Daniel Tuan-Dan Le	277	MWF	9:30AM	Iryna Egorova
202	TR	3:00PM	Ning Wei	281	TR	7:30AM	Arun Albert Debray
213	TR	4:30PM	Ning Wei	282	TR	12:00PM	Arun Albert Debray
214	MWF	12:30PM	Ping Xu	283	MWF	2:30PM	Oleksandr Tsymbaliuk
225	MWF	2:30PM	Ping Xu	284	TR	1:30PM	Jing Wang
226	MWF	1:30PM	Ping Xu	285	MWF	12:30PM	Yi Wang
237	MWF	10:30AM	Sai Kee Yeung	287	MWF	1:30PM	Yi Wang
238	MWF	3:30PM	Siamak Yassemi	288	MWF	7:30AM	Krishnendu Khan
240	MWF	12:30PM	Daniel Lentine Johnstone	289	MWF	8:30AM	Krishnendu Khan
241	MWF	11:30AM	Daniel Lentine Johnstone	298	MWF	4:30PM	Siamak Yassemi
252	TR	10:30AM	Raechel Polak	299	MWF	12:30PM	Ying Zhang
253	TR	9:00AM	Raechel Polak	300	MWF	1:30PM	Ying Zhang

4. There are 25 questions, each is worth 8 points. Show your work on the question sheets. also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

5. **Please remain seated during the last 10 minutes of the exam.** When time is called, all students must put down their writing instruments immediately.

6. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.

1. Find the value of a such that the following linear system has no solution.

$$\begin{cases} x + \;\; y - \;\; az = 1 \\ x + 2y + \;\; 2z = 3 \\ \qquad\; y + a^2 z = a \end{cases}$$

A. $a = 1$

B. $a = -1$

C. $a = 2$

D. $a = -2$

E. None of the above

2. For $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, compute ABC.

A. $ABC = \begin{bmatrix} 2 & 3 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$

B. $ABC = \begin{bmatrix} 1 & 4 \\ 4 & 6 \\ -6 & -5 \end{bmatrix}$

C. $ABC = \begin{bmatrix} 2 & 3 \\ -6 & -5 \\ 4 & 6 \end{bmatrix}$

D. $ABC = \begin{bmatrix} 1 & 4 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$

E. $ABC = \begin{bmatrix} 1 & 4 \\ -6 & -5 \\ 4 & 6 \end{bmatrix}$

3. It is known that the vectors

$$\begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} h \\ 0 \\ -1 \end{bmatrix}$$

become linear dependent when h takes values a or b, where $a \neq b$. What is $a + b$?

A. 3

B. 2

C. -4

D. -1

E. 0

4. Let A and B be $n \times n$ matrices. Which of the following statements are TRUE?

(i) If $\text{rank}\,(A) = n$ and $\text{Nullity}\,(B) = 0$, then $\text{rank}\,(AB) = 0$.

(ii) If the system $AB\mathbf{x} = 0$ has a unique solution, then $\text{rank}\,(B) = n$.

(iii) If the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, and the linear transformation $\mathbf{x} \mapsto B\mathbf{x}$ is onto, then the linear transformation $\mathbf{x} \mapsto AB\mathbf{x}$ is onto.

(iv) If the columns of A span $\mathbb{R}^n$, then $\text{rank}\,(AB) = \text{rank}\,(B)$.

(v) If $\text{rank}\,(AB) < n$, then $\text{rank}\,(A) < n$.

A. (i), (ii) and (iii) only

B. (ii), (iii), and (iv) only

C. (i), (iv) and (v) only

D. (iii) and (v) only

E. (ii) and (iv) only

5. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and A be the standard matrix of T. Let R be the row reduced echelon form for A. Which of the following statements is FALSE?

A. If $\text{Nul}\,(A) = \{\mathbf{0}\}$ and $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a linearly independent set in $\mathbb{R}^n$, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)\}$ is a linearly independent set in $\mathbb{R}^m$.

B. If R has a pivot in every row, then T is onto.

C. If $\text{rank}\,(A) < m$, then T is **not** one-to-one.

D. If $n = m$, then $A\mathbf{x} = \mathbf{0}$ only has the solution $\mathbf{x} = \mathbf{0}$ if and only if T is onto.

E. If T is one-to-one, then $m \geq n$.

6. Let $T \colon \mathbb{R}^2 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}.$$

Then $T\left(\begin{bmatrix} 1 \\ -4 \end{bmatrix}\right) =$

A. $\begin{bmatrix} -8 \\ 4 \\ 8 \end{bmatrix}$

B. $\begin{bmatrix} -2 \\ 4 \\ 12 \end{bmatrix}$

C. $\begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} -12 \\ 4 \\ 8 \end{bmatrix}$

E. $\begin{bmatrix} 10 \\ -6 \\ -6 \end{bmatrix}$

7. Suppose that $\det \begin{bmatrix} a & b & 3 \\ c & d & 4 \\ e & f & 3 \end{bmatrix} = 1$ and $\det \begin{bmatrix} a & b & 2 \\ c & d & 2 \\ e & f & 2 \end{bmatrix} = 2$. What is $\det \begin{bmatrix} a & b & 0 \\ c & d & 1 \\ e & f & 0 \end{bmatrix}$?

A. -2

B. -1

C. 0

D. 1

E. 2

8. Let $\mathbb{P}_3$ be the vector space of all polynomials of degree at most 3 and the zero polynomial. Which of the following subsets are subspaces of $\mathbb{P}_3$?

(i) The subset of polynomials $p(t) \in \mathbb{P}_3$ such that $p(1) = 0$.

(ii) The subset of polynomials $p(t) \in \mathbb{P}_3$ such that $p(0)p'(0) = 0$.

(iii) The subset of polynomials $p(t) \in \mathbb{P}_3$ with degree at most 1 and the zero polynomial.

(iv) The subset of polynomials $p(t) \in \mathbb{P}_3$ such that $p(0) \geq 0$.

(v) The subset of polynomials $p(t) \in \mathbb{P}_3$ with the coefficient of t^2 equal to 0.

A. (i) and (iii) only.

B. (ii), and (iii) only.

C. (i), (iii), and (v) only.

D. (ii) and (iv) only.

E. (i), (ii), (iv), and (v) only.

9. Consider the matrix

$$A = \begin{bmatrix} 3 & 3 & 3 \\ -2 & 2 & -3 \\ 4 & 2 & 4 \end{bmatrix}.$$

Let B be the inverse of the matrix A, and $B = [b_{ij}]$. Compute b_{12}.

A. 1

B. -1

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

E. $-\frac{1}{2}$

10. Suppose that V is a subspace spanned by $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$. Which of the following statements is NOT always true?

A. The dimension of V is at most p.

B. Any set of p linearly independent vectors in V is a basis for V.

C. Any set of $p+1$ vectors in V is linearly dependent.

D. If d is the dimension of V, then $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_d\}$ is a basis for V.

E. If $\mathbf{v}_p$ is in the span of the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{p-1}$, then $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{p-1}$ span V.

11. Consider the following 3×5 matrix:

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ 0 & 2 & 5 & -1 & -3 \\ 1 & 2 & 3 & -1 & 0 \end{bmatrix}.$$

Which of the following statements is FALSE?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis of Col A.

B. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ -1 \end{bmatrix} \right\}$ is a basis of Col A.

C. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 8 \\ 5 \\ 3 \end{bmatrix} \right\}$ is a basis of Col A.

D. $\left\{ \begin{bmatrix} 4 \\ -5 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 1 \\ 4 \\ -1 \end{bmatrix} \right\}$ is a basis of Nul A.

E. $\left\{ \begin{bmatrix} 4 \\ -5 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \\ 0 \\ 8 \\ -2 \end{bmatrix} \right\}$ is a basis of Nul A.

12. Given two 3×3 matrices A and B, compute $h = \det(A^2) - \det(BA^T B^{-1})$, where

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -7 & 4 & 2 \\ -7 & 3 & 2 \\ -3 & 1 & 1 \end{bmatrix}.$$

A. $h = 6$

B. $h = -6$

C. $h = -12$

D. $h = 12$

E. $h = 0$

13. Let H denote the subspace of the vector space $\mathbb{M}_{3\times 3}$ of all 3×3 matrices, consisting of A such that

$$A = A^T \qquad \text{and} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot A = A \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

The dimension of H is equal to:

A. 1

B. 2

C. 3

D. 4

E. 5

14. For the system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$ with $A = \begin{bmatrix} 3 & 5 \\ 4 & 4 \end{bmatrix}$, the origin is

A. a repeller

B. an attractor

C. a saddle point

D. a spiral point

E. none of the above

15. Let $\mathbb{M}_{2\times 2}$ be the vector space of 2×2 matrices, and let $T : \mathbb{M}_{2\times 2} \to \mathbb{M}_{2\times 2}$ be the linear transformation given by $T(A) = A + A^T$. Let $[T]_{\mathcal{B}}$ be the matrix for T relative to the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ for $\mathbb{M}_{2\times 2}$. What is the rank of $[T]_{\mathcal{B}}$?

A. 0

B. 1

C. 2

D. 3

E. 4

16. Let A be a 2×2 real matrix satisfying $A \begin{bmatrix} 2+i \\ 1-2i \end{bmatrix} = \begin{bmatrix} 7+i \\ 1-7i \end{bmatrix}$. Which of the following is an eigenvalue of A?

A. $1 + 2i$

B. $1 + i$

C. $2 + i$

D. $3 + i$

E. $3 + 2i$

17. Consider the following system of differential equations

$$x'(t) = 2x(t) + y(t)$$
$$y'(t) = 2x(t) + 3y(t)$$

with the initial condition $x(0) = 3, y(0) = 0$. What is the value of $x(1)$?

A. $3e^4$

B. $-2e + 2e^4$

C. $-2e - 2e^4$

D. $2e - e^4$

E. $2e + e^4$

18. Which of the following matrices is NOT diagonalizable over the real numbers?

A. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 3 & 5 \\ 3 & 0 & 0 \\ 5 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 7 \\ 0 & 0 & 7 \end{bmatrix}$

19. Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -1 \end{bmatrix}.$$

Which of the following is a basis $\mathcal{B}$ for $\mathbb{R}^2$ such that the matrix $[T]_{\mathcal{B}}$ for T relative to the basis $\mathcal{B}$ is a diagonal matrix?

A. $\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right\}$

20. Consider the vectors $\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $W = \text{Span}\{\mathbf{u}, \mathbf{v}\}$. Which of the following sets of vectors is $W^{\perp}$?

A. All vectors $\mathbf{x}$ satisfying $A^T\mathbf{x} = \mathbf{0}$

B. All vectors $\mathbf{x}$ satisfying $A\mathbf{x} = \mathbf{0}$.

C. All vectors $\mathbf{x}$ in $\mathbb{R}^3$ such that the matrix $\begin{bmatrix} \mathbf{x} & \mathbf{u} & \mathbf{v} \end{bmatrix}$ is invertible.

D. All vectors $\mathbf{x}$ in $\mathbb{R}^3$ such that the matrix $\begin{bmatrix} \mathbf{x} & \mathbf{u} & \mathbf{v} \end{bmatrix}$ is not invertible.

E. All vectors that are linear combinations of $\mathbf{u}$ and $\mathbf{v}$.

21. Let $\mathbf{y} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$. Find the distance from $\mathbf{y}$ to the plane in $\mathbb{R}^3$ spanned by $\mathbf{u}$ and $\mathbf{v}$.

A. $\dfrac{5\sqrt{3}}{3}$

B. $\dfrac{5}{3}$

C. $3\sqrt{3}$

D. $\dfrac{\sqrt{186}}{3}$

E. $\dfrac{\sqrt{3}}{3}$

22. Let A be an $n \times n$ matrix. Which of the following is **not** always true?

A. If A is invertible, and A^{-1} is orthogonally diagonoalizable, then A is orthogonally diagonalizable.

B. A is an orthogonal matrix if and only if A^{-1} exists and is orthogonal.

C. If $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $A^T A\mathbf{x} = A^T\mathbf{b}$ is inconsistent.

D. If A is diagonalizable, and 0 is not an eigenvalue of A, then A^{-1} exists and A^{-1} is diagonalizable.

E. Let $W = \mathrm{Col}\,(A)$. If $\mathbf{b} \in \mathbb{R}^n$ and $\mathbf{w} = \mathrm{proj}_W\mathbf{b}$, then $||\mathbf{b} - \mathbf{w}|| \leq ||\mathbf{b} - A\mathbf{x}||$ for all $\mathbf{x} \in \mathbb{R}^n$.

23. Find a least-squares solution of an inconsistent system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}.$$

A. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 1/2 \end{bmatrix}$

C. $\begin{bmatrix} -3 \\ 1/2 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ -1/2 \end{bmatrix}$

E. $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

24. Find a matrix P such that

$$P^{-1}\begin{bmatrix} 5 & 8 & -4 \\ 8 & 5 & -4 \\ -4 & -4 & -1 \end{bmatrix} P = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 15 \end{bmatrix}.$$

A. $P = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}$

B. $P = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

C. $P = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 2 \end{bmatrix}$

D. $P = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

E. $P = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

25. Consider the basis S for $\mathbb{R}^3$ given by $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$. If we apply the Gram–Schmidt process to S to obtain an orthonormal basis, we obtain

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 2/3 \\ 1/3 \end{bmatrix} \right\}$

Scratch paper

Scratch paper