MA26600 Exam 1

GREEN VERSION

NAME: ______________________________

PUID (10 digits): ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 8 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Find the explicit solution to the initial value problem

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{xy}{\sqrt{1+x^2}} \\ y(0) = 1 \end{cases}$$

A. $y = e^{1+x^2} - e + 1$

B. $y = e^{\sqrt{1+x^2}}$

C. $y = e^{\sqrt{1+x^2}-1}$

D. $y = xe^{\sqrt{1+x^2}} + 1$

E. $y = \sin(1 + x^3) + 1$

2. A tank contains 200 Liters of water and 40 grams of sugar. Water containing a sugar concentration of $5e^{-t}$ g/L flows into the tank at a rate of 4 L/min, and the mixture in the tank flows out at a rate of 6 L/min. Let $A(t)$ be the amount of sugar (in grams) in the tank at time t (in minutes). Which differential equation does $A(t)$ satisfy?

A. $\dfrac{dA}{dt} = 4e^{-t} - \dfrac{6A}{200 + 2t}$

B. $\dfrac{dA}{dt} = 4e^{-t} - \dfrac{6A}{200}$

C. $\dfrac{dA}{dt} = 5e^{-t} - \dfrac{6A}{40 + 2t}$

D. $\dfrac{dA}{dt} = 20e^{-t} - \dfrac{6A}{200 - 2t}$

E. $\dfrac{dA}{dt} = 20e^{-t} - \dfrac{6A}{40 - 2t}$

3. If y is a solution to the initial value problem for a Bernoulli equation

$$\frac{dy}{dx} + \frac{4y}{x} = 3y^2, \quad y(1) = \frac{1}{2},$$

then $y(2) = ?$ *Hint:* Use the substitution $v = y^{-1}$.

A. $\frac{1}{18}$

B. $\frac{1}{34}$

C. $\frac{1}{10}$

D. $\frac{1}{20}$

E. $\frac{1}{32}$

4. Use Euler's method with step size $h = \frac{1}{2}$ to find the approximate value of $y(1)$, where $y(x)$ is the solution to the initial value problem

$$y' = 4x + y, \quad y(0) = -8.$$

A. -12

B. -17

C. -8

D. $-\frac{47}{2}$

E. -10

5. If $y(x)$ is the solution to the initial value problem

$$y'' + y' - 2y = 0, \quad y(0) = 4, \quad y'(0) = 1,$$

then $y(1) + y'(1) =?$

A. $6e - e^{-2}$

B. $3e - e^{2}$

C. $6e$

D. $e - 3e^{-2}$

E. $-3e^{-2}$

6. Find a general solution of $y^{(4)} - 2y'' + y = 0$.

A. $y = \cos x(C_1 + C_2 x) + \sin x(C_3 + C_4 x)$

B. $y = (C_1 e^{x} + C_2 e^{-x})(C_3 + C_4 x)$

C. $y = C_1 + C_2 x + C_3 x^2 + C_4 x^3$

D. $y = C_1 e^{x} + C_2 e^{-x}$

E. $y = e^{-x}(C_1 + C_2 x) + e^{x}(C_3 + C_4 x)$

7. Find the value of b for which the given equation is exact, and then solve it using that value of b.

$$(y\cos(3xy) + b\,x)dx + (b\,x\cos(3xy) - 2y)dy = 0$$

A. $b = -2,\quad -\dfrac{2\sin(3xy)}{3} - y^2 - x^2 = C$

B. $b = 1,\quad \dfrac{\cos(3xy)}{9y^2} + \dfrac{x\sin(3xy)}{3y} - y^2 + xy = C$

C. $b = 3,\quad \sin(3xy) - y^2 + \dfrac{3x^2}{2} = C$

D. $b = 1,\quad \dfrac{\sin(3xy)}{3} - y^2 + \dfrac{x^2}{2} = C$

E. $b = 3,\quad \dfrac{\sin(3xy)}{3} - y^2 + \dfrac{3x^2}{2} = C$

8. Given that $y = x^r$ satisfies the following equation

$$2x^2y'' + xy' - 3y = 0, \qquad x > 0.$$

(a) Find all the possible values of r.

(b) Find a general solution of the differential equation.

9. A population's size at time t is $x(t)$ and is modeled by the differential equation

$$\frac{dx}{dt} = \frac{1}{3}x(x-3). \tag{$*$}$$

(a) Sketch a phase diagram for the differential equation in $(*)$. What does this phase diagram tell you about $\lim_{t\to\infty} x(t)$ if $x(0) = 1$?

(b) Find an explicit solution to $(*)$ when $x(0) = 1$.

10. A mass of 2 kg is attached to the end of a spring that is stretched 2 m by a force of 100 N. At time $t = 0$ s, the mass is set in motion from its equilibrium position with initial velocity $v_0 = 20$ m/s.

(a) Let $x(t)$ be the displacement (in meters) of the mass at time t (in seconds). Write down an initial value problem for $x(t)$.

(b) Solve the initial value problem from part (a). What are the amplitude and frequency of $x(t)$?