MA26600 Exam 1

GREEN VERSION

NAME: _____

PUID (10 digits): _____

INSTRUCTOR: _____

SECTION/TIME: _____

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 9 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

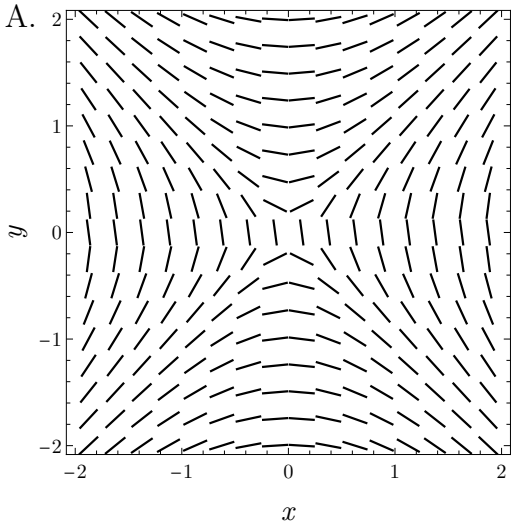
   (D.) My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

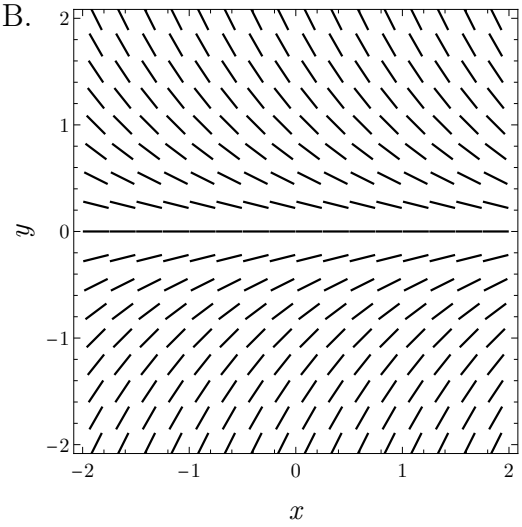
   Turn off or put away all electronic devices.

1. Identify the slope field of the differential equation $y' = -xy$.

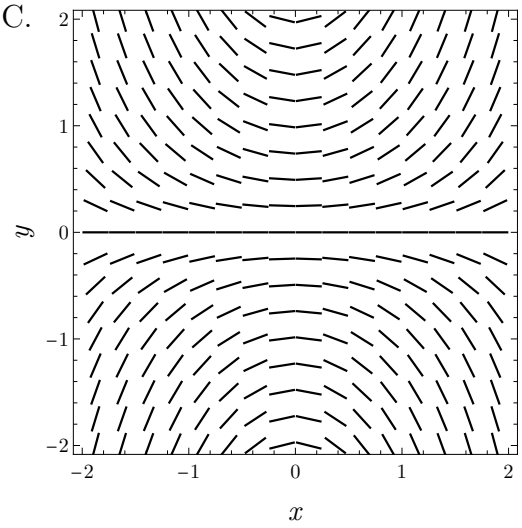
A.



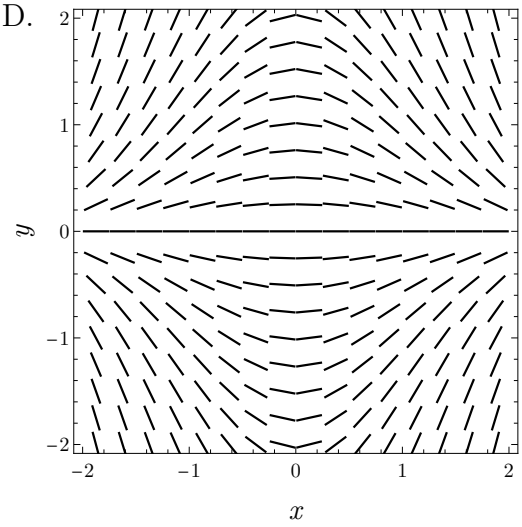
B.



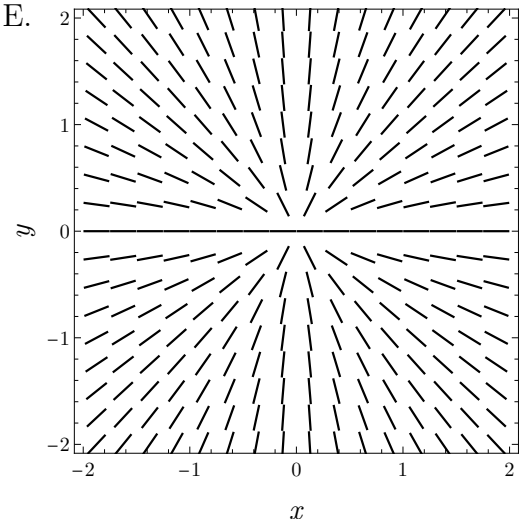
C.



D.



E.



2. Solve the following initial value problem by using the separation of variables.

$$\frac{dy}{dx} = \frac{4x^2y^2 + y^2}{x^2y^2 + x^2}, \qquad y(-1) = 1.$$

A. $\frac{1}{3}y^3 + y - \frac{4}{3}x^3 - x = \frac{11}{3}$

B. $y - \frac{1}{y} - 4x + \frac{1}{x} = 3$

C. $\frac{1}{3}y^3 + y + \frac{4}{3}x^3 + x = -1$

D. $4y - \frac{1}{y} - x + \frac{1}{x} = 3$

E. $y + \frac{1}{y} - 4x - \frac{1}{x} = 7$

3. Find the general solution to the differential equation

$$e^x \frac{dy}{dx} + 2y = 2e^{x+2e^{-x}}.$$

A. $y = x^2 e^{2e^{-x}} + Ce^{2e^{-x}}$

B. $y = xe^{x+2e^{-x}} + Ce^{x+2e^{-x}}$

C. $y = x^2 e^x + Ce^x$

D. $y = x^2 e^{-x} + Ce^{x+2e^{-x}}$

E. $y = 2xe^{2e^{-x}} + Ce^{2e^{-x}}$

4. Find the general solution to the homogenous differential equation using the substitution $v = y/x$. Assume $x > 0$.

$$x^2 y' = xy + x^2 e^{y/x}.$$

A. $y = -x \ln(C - \ln x)$

B. $y = x(\ln x + C)$

C. $y = -\ln(C - \ln x)$

D. $y = x \ln\left(\dfrac{x^2}{2} + C\right)$

E. $y = -x \ln(C - x)$

5. Consider a population of fish that is regularly harvested and has population $x(t)$ which is modeled by the "Logistic-with-harvest" equation $x' = x(6-x) - 8$. Which of the following is true about the critical points of the system?

A. There is a stable point at $x = 0$ and an unstable point at $x = 6$.

B. There is an unstable point at $x = 2$ and a stable point at $x = 4$.

C. There is a stable point at $x = 2$ and a semistable point at $x = 4$.

D. There are no critical points of the system.

E. There is only a stable point at $x = 6$.

6. Apply Euler's method with three steps of size $h = 1/3$ to approximate the value $y(1)$ of the solution to

$$y' = 18ty - 1, \qquad y(0) = 0.$$

A. $-3/4$

B. $-11/9$

C. -21

D. $-421/3$

E. -7

7. Find a differential equation whose general solution is

$$y(t) = c_1 e^{2t} + c_2 e^{t} + c_3 e^{-t}.$$

A. $y''' + 2y'' - y' - 2y = 0$

B. $-2y''' + 4y'' - 2y' - 4y = 0$

C. $y''' - 2y'' - y' + 2y = 0$

D. $y''' - 2y'' + y' + 2y = 0$

E. $-y''' + 2y'' - y' - 2y = 0$

8. Solve the initial value problem

$$y'' + 6y' + 9y = 0, \qquad y(0) = 1,\ y'(0) = 1.$$

9. Find the general solution of the Bernoulli equation below using the substitution $v = y^{-2}$.

$$y' - y = 4y^3, \qquad y > 0.$$

10. A ball with mass $m = 1$ kg is shot upward from the ground level with the initial velocity $v(0) = v_0$. It is subject to the Earth's gravitational acceleration $g = 9.8$ m/s^2. Air resistance is modeled by a force $k|v|$ opposite to the velocity, with $k = 2$ kg/s.

(a) Compute the velocity $v(t)$ of the ball before it reaches its maximum height.

(b) Suppose the ball reaches its maximal height at time $t_0 = \ln 2$ seconds. Show that $v_0 = 14.7$ m/s.