MA26600 Exam 2

GREEN VERSION

NAME: ____________________

PUID (10 digits): ____________________

INSTRUCTOR: ____________________

SECTION/TIME: ____________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 8 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Set up an appropriate form for the particular solution $y_p(x)$, but do not determine the values of the coefficients. (All derivatives in this problem are with respect to x).

$$y^{(3)} - y'' - 2y' = x^2 + e^{2x}$$

A. $y_p(x) = Ax^2 + Bx + C + De^{2x}$

B. $y_p(x) = Ax^3 + Bx^2 + Cx + Dxe^{2x}$

C. $y_p(x) = Ax^2 + Be^{2x}$

D. $y_p(x) = A + Be^{2x} + Ce^{-x}$

E. $y_p(x) = Ax^2 + Bx + C + (Dx + E)e^{2x}$

2. Solve the initial value problem

$$\begin{cases} x'(t) = x(t) - 4y(t) \\ y'(t) = -2x(t) + 8y(t), \end{cases}$$

where $x(0) = 5$ and $y(0) = -1$.

A. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{9t} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{t}$

B. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{9t} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{t}$

C. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{9t} + \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{9t}$

E. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{9t} + \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

3. Consider the following system of equations

$$\begin{aligned} x_1' &= x_1 + x_2 + x_3 \\ x_2' &= x_1 - 2x_2 + 4x_3 \\ x_3' &= x_1 + 4x_2 - 2x_3. \end{aligned}$$

Assume that for some non-zero constant vector $\mathbf{v}$, $\mathbf{x}(t) = e^{3t}\,\mathbf{v}$ is a solution to the above system. Which of the following vectors can $\mathbf{v}$ equal?

A. $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ B. $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ D. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ E. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

4. For the linear system

$$\mathbf{x}' = \begin{bmatrix} 0 & 1 \\ c & -2 \end{bmatrix} \mathbf{x},$$

find all values of the parameter c such that the origin is a spiral sink.

A. $c < -1$

B. $-1 < c < 0$

C. $c > 0$

D. $c = -1$

E. $c = 0$

5. A spring-mass system is governed by the differential equation

$$2x'' + 72x = 100\sin(3\omega t)$$

For what value of ω will resonance occur?

A. 3

B. $6\sqrt{2}$

C. 2

D. 10

E. No value

6. The first order linear system

$$\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = -t^2 x_1 + 2t x_2 + 2x_3 + e^t \end{cases}$$

is equivalent to which of the following differential equations.

A. $x''' + t^2 x'' - 2tx' - 2x = e^t$

B. $x''' - 2x'' - 2tx' + t^2 x = e^t$

C. $x^{(4)} + t^2 x''' - 2tx'' - 2x' - e^t x = 0$

D. $-x^2 x' + 2xx'' + 2x''' + e^x = 0$

E. None of the above

7. A general solution of $\mathbf{x}' = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix} \mathbf{x}$ is

A. $c_1 e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 t e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

B. $c_1 e^{t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

C. $c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{-t} \left(t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} \right)$

D. $c_1 e^{t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 t e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

E. $c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{-t} \left(t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix} \right)$

8. Use variation of parameters to find a particular solution to

$$ty'' - (t+1)y' + y = t^2, \qquad (t > 0),$$

given that $y_1 = e^t$, $y_2 = t + 1$ are solutions of the corresponding homogeneous equation. (*Hint.* Rewrite the equation in the standard form first.)

9. Find a real-valued general solution of the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where

$$\mathbf{A} = \begin{bmatrix} 5 & -9 \\ 2 & -1 \end{bmatrix}.$$

10. Find a general solution of $x'' + 3x' + 2x = 10\cos t$ by using the method of undetermined coefficients.