MA26600 Final Exam

GREEN VERSION 01

NAME: ______________________________

INSTRUCTOR: ____________________ SECTION/TIME: __________

- You must use a **#2 pencil** on the mark-sense answer sheet.
- Fill in the **ten digit PUID** (starting with two zeroes) and your **Name** and blacken in the appropriate spaces.
- Fill in the correct **Test/Quiz number** (GREEN is **01**, ORANGE is **02**)
- Fill in the **four digit section number** of your class and blacken the numbers below them. Here they are:

0313	MWF	2:30PM	Shuyi Weng
0314	MWF	3:30PM	Shuyi Weng
0325	MWF	9:30AM	Al Volkening
0326	MWF	8:30AM	Al Volkening
0337	MWF	1:30PM	Moongyu Park
0338	MWF	11:30AM	Moongyu Park
0340	MWF	3:30PM	Moongyu Park
0341	MWF	8:30AM	Zhiyuan Geng
0352	MWF	9:30AM	Zhiyuan Geng
0353	MWF	9:30AM	Ying Liang
0364	MWF	10:30AM	Ying Liang
0365	TR	12:00PM	Heejin Lee
0376	TR	10:30AM	Heejin Lee
0901	TR	3:00PM	Jiahao Zhang
0902	TR	1:30PM	Jiahao Zhang
0903	TR	4:30PM	Guang Yang
0904	TR	12:00PM	Guang Yang
0905	MWF	8:30AM	Jeaheang Bang
0906	MWF	9:30AM	Jeaheang Bang
0915	MWF	9:30AM	Gayane Poghotanyan
0916	MWF	10:30AM	Gayane Poghotanyan

- Sign the mark-sense sheet.
- Fill in your name and your instructor's name and the time of your class meeting on the exam booklet above.
- There are 20 multiple-choice questions, each worth 10 points. **Blacken in** your choice of the correct answer in the spaces provided for questions 1–20 in the answer sheet. Do all your work on the question sheets, in addition, also **Circle** your answer choice for each problem on the question sheets in case your mark-sense sheet is lost.
- Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
- **No calculators, books, electronic devices, or papers are allowed.** Use the back of the test pages for scratch paper.
- Pull off the **table of Laplace transforms** on the last page of the exam for reference. Do not turn it in with your exam booklet at the end.

1. Let $y(t)$ be the solution of the initial value problem

$$y\frac{dy}{dx} = x(y^2 + 1), \quad y(0) = 1.$$

Find the value of $y(2)$.

A. $y(2) = \sqrt{e^4 + 2}$

B. $y(2) = 2e^4 - 1$

C. $y(2) = \sqrt{2e^4 - 1}$

D. $y(2) = e^2 - 1$

E. $y(2) = \sqrt{2e^4}$

2. What is the largest open interval in which the solution of the initial value problem

$$(t-3)y' + \frac{\sqrt{t-1}}{6}y = \frac{2}{t-10}, \quad y(5) = 2$$

is guaranteed to exist by the Existence and Uniqueness Theorem?

A. $(1, 10)$

B. $(3, 10)$

C. $(1, 3)$

D. $(10, \infty)$

E. All real numbers except 10

3. If the Wronskian $W(f,g) = 4t^4$ and $f(t) = t^2$, for $t > 0$, then the function g could be

A. $Ct + 4t^4$

B. $Ct^2 + 4t^2$

C. $Ct^2 + 4t^4$

D. $Ct + 4t^3$

E. $Ct^2 + 4t^3$

4. A tank of total volume 200 L initially contains 100 L of pure water. A brine solution of concentration 0.2 kg/L is pumped into the tank at a rate of 10 L/s, and the well-mixed solution in the tank is pumped out at a rate of 7 L/s. Write the initial value problem for the amount of salt $x(t)$ (in kg) at time t (in s) before the tank overflows.

A. $\dfrac{dx}{dt} = 2 - \dfrac{7x}{100};\ x(0) = 0$

B. $\dfrac{dx}{dt} = 0.2 - \dfrac{x}{100 + 3t};\ x(0) = 0$

C. $\dfrac{dx}{dt} = 10 - \dfrac{7x}{100};\ x(0) = 0$

D. $\dfrac{dx}{dt} = 2 - \dfrac{7x}{100 + 3t};\ x(0) = 0$

E. $\dfrac{dx}{dt} = 2 - \dfrac{7x}{200 - 3t};\ x(0) = 100$

5. Find the number of the stable critical points for the autonomous equation

$$\frac{dx}{dt} = x(x+1)^2(x-3)(x^2-4).$$

A. 2

B. 1

C. 3

D. 4

E. 0

6. Solve the initial value problem

$$y'' - 10y' + 26y = 0, \quad y(0) = 1,\ y'(0) = 4.$$

A. $y(t) = e^t \left(\cos 5t - \frac{2}{5} \sin 5t \right)$

B. $y(t) = e^{5t}(\cos t + 4 \sin t)$

C. $y(t) = e^t \left(\cos 5t + \frac{3}{5} \sin 5t \right)$

D. $y(t) = e^{5t}(\cos t + 2 \sin t)$

E. $y(t) = e^{5t}(\cos t - \sin t)$

7. Given that $y_1 = x$ and $y_2 = \dfrac{1}{x}$ are two solutions to a second-order linear differential equation

$$y'' + P(x)y' + Q(x)y = 0, \quad x > 0,$$

which one of the following functions is a solution to

$$y'' + P(x)y' + Q(x)y = \frac{2}{x^2}, \quad x > 0 \ ?$$

A. $y(x) = \dfrac{2}{x}$

B. $y(x) = -2$

C. $y(x) = -\dfrac{2}{x}$

D. $y(x) = -x^2$

E. $y(x) = -\dfrac{1}{x^2}$

8. Find the general solution of $y^{(3)} + 6y'' + 9y' = 0$.

A. $y(t) = c_1 + c_2 e^{-3t} + c_3 t e^{-3t}$

B. $y(t) = c_1 t + c_2 e^{3t} + c_3 t e^{3t}$

C. $y(t) = c_1 + c_2 \cos(-3t) + c_2 \sin(-3t)$

D. $y(t) = c_1 t + c_2 e^{-3t} + c_3 e^{3t}$

E. $y(t) = c_1 t + c_2 \cos(3t) + c_2 \sin(3t)$

9. Given that the characteristic polynomial of the homogeneous differential equation $y^{(6)} - 6y^{(5)} + y^{(4)} + 54y''' - 90y'' = 0$ is

$$P(r) = (r^4 - 9r^2)(r^2 - 6r + 10)\,,$$

use the method of undetermined coefficients to find the form of a particular solution to

$$y^{(6)} - 6y^{(5)} + y^{(4)} + 54y''' - 90y'' = 1 + 4xe^{3x} - \sin 3x\,.$$

A. $y_p = A + Bx^2e^{3x} + Cxe^{3x} + D\sin 3x + E\cos 3x$

B. $y_p = Ax + Bxe^{3x} + Ce^{3x} + D\cos 3x + E\sin 3x$

C. $y_p = Ax^2 + Bx^3e^{3x} + Cx^2e^{3x} + D\sin 3x + E\cos 3x$

D. $y_p = Ax^2 + Bx^2e^{3x} + Cxe^{3x} + D\cos 3x + E\sin 3x$

E. $y_p = Ax^2 + Bx + Cx^2e^{3x} + Dxe^{3x} - E\cos 3x$

10. For the initial value problem,

$$y' = x + 2y, \quad y(0) = 3,$$

find the approximation of $y(2)$ using Euler's method with step size $h = 1$.

A. 3

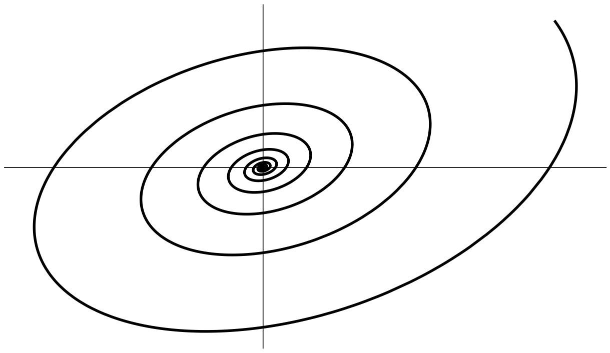
B. -13

C. 7

D. 19

E. 28

11. Given the phase portrait of the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, what are the possible eigenvalues of the matrix $\mathbf{A}$?



A. $\pm 5i$

B. $-5, 3$

C. $-1 \pm 10i$

D. $-7, -2$

E. $1, 5$

12. Find the matrix exponential $e^{\mathbf{A}t}$ for the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

A. $\begin{bmatrix} e^{2t} & e^{-t} \\ e^{2t} & -e^{-t} \end{bmatrix} \begin{bmatrix} -1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$

B. $\begin{bmatrix} e^{3t} & e^{-t} \\ e^{3t} & -e^{-t} \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$

C. $\begin{bmatrix} e^{3t} & e^{-t} \\ e^{3t} & -e^{-t} \end{bmatrix} \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & -1/2 \end{bmatrix}$

D. $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} e^{3t} & e^{-t} \\ e^{3t} & -e^{-t} \end{bmatrix}$

E. $\begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} e^{3t} & e^{-t} \\ e^{3t} & -e^{-t} \end{bmatrix}$

13. Consider the initial value problem

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}.$$

Find $x_1(t)$.

A. $-e^{-2t} + 2e^{-4t}$

B. e^{-2t}

C. $2e^{-2t} - e^{-4t}$

D. $3e^{-t} - 2e^{-6t}$

E. $-2e^{-2t} + 3e^{-4t}$

14. Which of the following is a general solution of the linear system

$$\mathbf{x}' = \begin{bmatrix} -6 & -1 \\ 4 & -2 \end{bmatrix} \mathbf{x} \ ?$$

A. $\mathbf{x}(t) = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} t \\ 1 - 2t \end{bmatrix}$

B. $\mathbf{x}(t) = c_1 e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} t \\ -1 + 2t \end{bmatrix}$

C. $\mathbf{x}(t) = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} t \\ -1 - 2t \end{bmatrix}$

D. $\mathbf{x}(t) = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ -t - 2 \end{bmatrix}$

E. $\mathbf{x}(t) = c_1 e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} t \\ 1 + 2t \end{bmatrix}$

15. If we use the method of undetermined coefficients to find a particular solution of

$$\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

what is the appropriate form for a trial solution?

A. $\mathbf{x}(t) = \begin{bmatrix} A \\ B \end{bmatrix} + \begin{bmatrix} C \\ D \end{bmatrix} e^{-t}$

B. $\mathbf{x}(t) = \begin{bmatrix} A \\ B \end{bmatrix}$

C. $\mathbf{x}(t) = \begin{bmatrix} A \\ B \end{bmatrix} t$

D. $\mathbf{x}(t) = \begin{bmatrix} A \\ B \end{bmatrix} t + \begin{bmatrix} C \\ D \end{bmatrix} + \begin{bmatrix} E \\ F \end{bmatrix} e^{-t}$

E. $\mathbf{x}(t) = \begin{bmatrix} A \\ B \end{bmatrix} t + \begin{bmatrix} C \\ D \end{bmatrix}$

16. The inverse Laplace transform of $F(s) = \dfrac{s+1}{s^2+s+1/4}$ is

A. $e^{-t/2}$

B. $e^{t/2} + \frac{t}{2}e^{t/2}$

C. $e^{-t/2} + \frac{t}{2}e^{-t/2}$

D. $e^{-t/2} + \frac{3t}{2}e^{-t/2}$

E. $e^{-t/2} - \frac{t}{2}e^{-t/2}$

17. The solution $y(t)$ for the initial value problem

$$y'' + 2y' + 5y = \delta(t-5), \quad y(0) = y'(0) = 0$$

for $t > 5$ is:

A. $\frac{1}{2}e^{-t}\sin(2(t-5))$

B. $\frac{1}{2}e^{-2(t-5)}\sin(2(t-5))$

C. $\frac{1}{2}\sin(2(t-5))$

D. $\frac{1}{2}e^{-(t-5)}\sin(2(t-5))$

E. $\frac{1}{2}e^{-t}\sin(2t)$

18. Find the Laplace transform $F(s)$ of $f(t) = 2t\sin(3t)$.

A. $F(s) = \dfrac{12s}{(s^2+9)^2}$

B. $F(s) = \dfrac{-12s}{(s^2+9)^2}$

C. $F(s) = \dfrac{6}{s^2(s^2+9)}$

D. $F(s) = \dfrac{12}{(s+1)^2+9}$

E. $F(s) = \dfrac{12}{(s^2+9)^2}$

19. Find the Laplace transform of $f(t) = \begin{cases} 1, & \text{if } 0 \le t < 2, \\ t^2, & \text{if } 2 \le t. \end{cases}$

A. $\dfrac{1}{s} + e^{-2s}\left(\dfrac{2}{s^3} - \dfrac{4}{s^2} + \dfrac{4}{s}\right)$

B. $\dfrac{1}{s} + e^{-2s}\left(\dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{3}{s}\right)$

C. $\dfrac{1}{s} + e^{-2s}\dfrac{2}{s^3}$

D. $\dfrac{1}{s} + e^{-2s}\left(\dfrac{2}{s^3} - \dfrac{1}{s}\right)$

E. $\dfrac{1}{s} + e^{-2s}\left(\dfrac{2}{s^3} - \dfrac{5}{s^2} + \dfrac{6}{s}\right)$

20. The solution of the initial value problem

$$y'' + 3y' + 2y = g(t), \quad y(0) = 2,\ y'(0) = -4.$$

is given by:

A. $e^{-2t} + 2\displaystyle\int_0^t g(t-\tau)(e^{-\tau} - e^{-2\tau})d\tau$

B. $2e^{-t} + 4e^{-2t} + \displaystyle\int_0^t g(t-\tau)(e^{-\tau} - e^{-2\tau})d\tau$

C. $\sin(2t) + \displaystyle\int_0^t g(t-\tau)e^{-2\tau}d\tau$

D. $2e^{-2t} + \displaystyle\int_0^t g(t-\tau)(e^{-\tau} - e^{-2\tau})d\tau$

E. $\cos(2t) + \displaystyle\int_0^t g(t-\tau)e^{-\tau}d\tau$

Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}, \quad s > 0$
2.	e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3.	$t^n, \quad n =$ positive integer	$\dfrac{n\,!}{s^{n+1}}, \quad s > 0$
4.	$t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \|a\|$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \|a\|$
9.	$e^{at} \sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10.	$e^{at} \cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11.	$t^n e^{at}, \quad n =$ positive integer	$\dfrac{n\,!}{(s-a)^{n+1}}, \quad s > a$
12.	$u(t-c)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13.	$u(t-c)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(c\,t)$	$\dfrac{1}{c}\,F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$t^nf(t)$	$(-1)^nF^{(n)}(s)$