

MA 16500
EXAM 1 INSTRUCTIONS
VERSION 01
September 18, 2024

Your name _____ Your TA's name _____

Student ID # _____ Section # and recitation time _____

1. You must use a #2 pencil on the scantron sheet (answer sheet).
2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.
3. On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.
4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.
5. Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.
6. **Sign the scantron sheet.**
7. Blacken your choice of the correct answer in the space provided for each of the questions 1–12. While marking all your answers on the scantron sheet, you should show your work on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.
8. There are 12 questions, 10 of which are worth 8 points and 2 of which are worth 10 points. The maximum possible score is
$$10 \text{ questions} \times 8 \text{ points} + 2 \text{ questions} \times 10 \text{ points} = 100 \text{ points.}$$
9. **NO calculators, electronic device, books, or papers are allowed (unless you have the special accomodation which permits their use).**
Use the back of the test pages for scrap paper.
10. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.
11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. (8 points) Solve for x in the following equations:

(i) $\ln x + \ln(x - 5) = 0$

(ii) $\sin x = \sin(2x)$ with condition $0 \leq x < 2\pi$.

A. (i) $\frac{5 \pm \sqrt{29}}{2}$ (ii) $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

B. (i) $\frac{5 \pm \sqrt{29}}{2}$ (ii) $\left\{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$

C. (i) $\frac{5 + \sqrt{29}}{2}$ (ii) $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

D. (i) $\frac{5 + \sqrt{29}}{2}$ (ii) $\left\{ 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3} \right\}$ (CORRECT)

E. (i) $0, 5$ (ii) $\{0\}$

2. (8 points) Consider the function

$$f(x) = \frac{x+3}{x-2}$$

over the specified domain $(0, \infty)$.

Find

- (i) the formula for the inverse function $f^{-1}(x)$, and
- (ii) the domain of the inverse function.

NOTE: The domain of the inverse function $f^{-1}(x)$ is the range of the original function $f(x)$ with its specified domain $(0, \infty)$.

- A. (i) $f^{-1}(x) = \frac{2x+3}{x-1}$ (ii) $(-\infty, -\frac{3}{2})$
- B. (i) $f^{-1}(x) = \frac{2x+3}{x-1}$ (ii) $(-\infty, -\frac{3}{2}) \cup (1, \infty)$ (CORRECT)
- C. (i) $f^{-1}(x) = \frac{2x+3}{x-1}$ (ii) $(1, \infty)$
- D. (i) $f^{-1}(x) = \frac{x-2}{x+3}$ (ii) $(0, \infty)$
- E. (i) $f^{-1}(x) = \frac{2x+3}{x-1}$ (ii) $(-\infty, 1)$

3. (8 points) Evaluate the limits

$$(i) \lim_{x \rightarrow 0} \frac{x}{\sin(9x)} \quad \text{and} \quad (ii) \lim_{x \rightarrow +\infty} \frac{\cos(x)}{3x}$$

- A. (i) $1/9$ (ii) 0 (CORRECT)
- B. (i) $1/9$ (ii) $1/3$
- C. (i) 9 (ii) DNE
- D. (i) 9 (ii) 0
- E. (i) DNE (ii) 0

4. (8 points) Evaluate $\lim_{x \rightarrow +\infty} (\sqrt{9x^2 - x} - 3x)$

A. $-\frac{1}{6}$ (CORRECT)

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. 0

E. ∞

5. (8 points) Consider the following function:

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Find the correct statement about continuity/differentiability of the function at $x = 0$.

- A. continuous and differentiable
- B. continuous but not differentiable (CORRECT)
- C. not continuous but differentiable
- D. not continuous and not differentiable
- E. continuous but we cannot determine whether it is differentiable or not

6. (8 points) Consider the following equation

$$x^3 + 2x = 5x^2 - 1$$

between -1 and 3 . Among the open intervals $(-1, 0)$, $(0, 1)$, $(1, 2)$, $(2, 3)$, choose the right combination which exhausts ALL containing a solution of the equation.

- A. $(0, 1)$, $(1, 2)$
- B. $(-1, 0)$, $(0, 1)$ (CORRECT)
- C. $(-1, 0)$, $(1, 2)$, $(2, 3)$
- D. $(0, 1)$, $(2, 3)$
- E. $(-1, 0)$, $(1, 2)$

7. (8 points) Suppose we have a function $f(x)$ whose derivative at $x = 4$ is equal to 6.

That is to say, we have

$$f'(4) = \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h} = 6.$$

Then evaluate the following limit

$$\lim_{h \rightarrow 0} \frac{f(4+5h) - f(4-2h)}{h}.$$

- A. $\frac{2}{3}$
- B. 7
- C. 24
- D. 42 (CORRECT)
- E. We cannot determine the value of the limit from the given information.

8. (8 points) Suppose that f, g and h are functions that satisfy the following conditions:

$$\begin{cases} f(0) = 2, f'(0) = 3, f'(-3) = 2, f(-3) = 0, \\ g(0) = -1, g'(0) = 5, \\ h(0) = 1, h'(0) = -2. \end{cases}$$

Evaluate the derivative of the following function at $x = 0$

$$F(x) = \frac{f^{-1}(x)}{g(x) - h(x)}.$$

- A. $F'(0) = 1$
- B. $F'(0) = 2$
- C. $F'(0) = 3$
- D. $F'(0) = 4$
- E. $F'(0) = 5$ (CORRECT)

9. (8 points) Find the formula for (i) and the exact value for (ii).

$$(i) \cos \left(\sin^{-1} \left(\frac{x}{3} \right) \right) \quad (ii) \sin^{-1} \left(\cos \left(\frac{3\pi}{5} \right) \right).$$

A. (i) $\frac{\sqrt{9-x^2}}{3}$ (ii) $-\frac{2\pi}{5}$

B. (i) $\frac{\sqrt{9-x^2}}{3}$ (ii) $-\frac{\pi}{10}$ (CORRECT)

C. (i) $\frac{\sqrt{9+x^2}}{3}$ (ii) $\frac{\pi}{10}$

D. (i) $\frac{3}{\sqrt{9+x^2}}$ (ii) $-\frac{\pi}{10}$

E. (i) $\frac{3}{\sqrt{9-x^2}}$ (ii) $-\frac{2\pi}{5}$

10. (8 points) Identify the number of
(V) VERTICAL asymptote(s), and
(H) HORIZONTAL asymptote(s)
for the graph of

$$y = f(x) = \frac{7 + e^x}{4 - e^x}.$$

- A. (V) 1 (H) 0
- B. (V) 1 (H) 1
- C. (V) 1 (H) 2 (CORRECT)
- D. (V) 2 (H) 1
- E. (V) 2 (H) 2

11. (10 points) Find an equation for the line tangent to the graph of $f(x) = \tan(x)$ over the interval $(-\pi/2, 0)$, which is parallel to the line $y = 2x + 1$.

A. $y = 2x + \frac{\pi}{2} - 1$. (CORRECT)

B. $y = 2x - \frac{\pi}{2} + 1$.

C. $y = 2x + \frac{\pi}{3} - \sqrt{3}$.

D. $y = 2x - \frac{\pi}{3} + \sqrt{3}$.

E. $y = 2x + \frac{\pi}{6} - \frac{\sqrt{3}}{3}$.

12. (10 points) Find the values of a and b so that the function

$$f(x) = \begin{cases} ax + 2b + 18 & \text{if } x \leq 2, \\ \frac{x^2 + ax + b}{x^2 - 4} & \text{if } x > 2 \end{cases}$$

is continuous over $(-\infty, \infty)$.

- A. $a = 5$ and $b = -10$
- B. $a = 7$ and $b = -14$
- C. $a = 4$ and $b = -12$ (CORRECT)
- D. $a = -12$ and $b = 4$
- E. One cannot determine the values of a and b from the given information.