

Answer Key to Green Exam 2

1. **B**

2. **A**

3. **C**

4. **E**

5. **D**

6. **B**

7. **D**

8. (1) $T(p(t)) = \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}$

(2) A basis for the range of T is $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. Answer may vary!

(3) A basis for the kernel of T is $\{t^3 - 4t, t^2 + 2t\}$. Answer may vary!

9. (1) $P = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$.

(2) $A^{2024} + 2A^{2025} = \begin{bmatrix} 5 & -3 \\ 10 & -6 \end{bmatrix}$.

10. (1) $\lambda_1 = 1, \quad \lambda_2 = 2024$

(2) $c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2024 \\ -1 \end{bmatrix} e^{2024t}$

(3) $c_1 = 1, \quad c_2 = 1, \quad x(1) + y(1) = 2023e^{2024}$