MATH 181, Final Exam

(25) **1.** Differentiate $\displaystyle\int_{-x}^{x} \frac{(t+1)^2}{1+t^5}\,dt$ with respect to x.

(50) **2.** Compute the following integrals.

a) $\displaystyle\int \frac{x^3}{(1+x^2)^{1/3}}\,dx$ Hint: let $u = 1 + x^2$.

b) $\displaystyle\int \frac{1}{(1+x^2)^2}\,dx$ Hint: let $x = \tan\theta$.

c) $\displaystyle\int x^2 \ln x\,dx$

d) $\displaystyle\int \frac{x-2}{x^2-4x+8}\,dx$

e) $\displaystyle\int xe^{-2x}\,dx$

(25) **3.** Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ for x where the series converges.

a) For what values of x does the series converge? Find all of them and explain your reasoning.

b) Identify the function $f(x)$ in non-series form.

(30) **4.** Find the solution to

$$\frac{dy}{dx} = y(1+y)$$

that satisfies the initial condition $y(0) = 2$.

(30) **5.** Let $S = \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$. Find a number N, with proof, such that

$$\left| S - \sum_{n=1}^{N} \frac{1}{n^4} \right| < 10^{-3}.$$

Hint: Compare the error, $\displaystyle\sum_{n=N+1}^{\infty} \frac{1}{n^4}$, to an integral. Be sure to draw a proper graph along with your explanation.

(40) **6.** Suppose $f(0) = 0$ and $f(2) = 0$, but $f(1) = 5$. Neither the first nor the second derivatives of f can have a small absolute value throughout $(0, 2)$. Make this statement precise.