MATH 425, Exam I

(25) **1.** Suppose that v_1 and v_2 are both harmonic conjugates for the same harmonic function u on a domain Ω. Show that v_1 and v_2 must differ by a constant on Ω.

(25) **2.** Compute

$$\int_\gamma \frac{1}{z}\, dz$$

where γ is any curve that starts at $1+i$ and ends at $-i$ and avoids the subset of the real axis $[0, \infty)$. Explain.

(25) **3.** Suppose $f(z)$ is an entire function. Show that if $\operatorname{Re} f(z) < 0$ for all z, then f must be a constant function.
Hint: Consider $\exp(f(z))$.

(25) **4.** Compute

$$\int_\gamma \frac{e^{iz}}{z^2+1}\, dz$$

where

a) γ is the counterclockwise circle of radius two about the origin.

b) γ is the *clockwise* circle of radius one about i.

c) γ is the counterclockwise circle of radius one-half about the origin.