Homework 9

1. Explain how to use e^{-x^4} to construct a "good" kernel on the real line.

2. Given that $f(x) = \dfrac{2}{\pi}\displaystyle\int_0^\infty \frac{\cos sx}{1+s^2}\,ds$, what is the Fourier transform of $f(x)$? Explain.

Hint: Even, odd functions. Inverse Fourier Transform.

3. Stein p. 91: Problem 14.

4. In HWK 7, Problem 1, you showed that

$$\alpha(w) = \int_{-\infty}^{\infty} g(x) \sin wx \; dx$$

goes to zero as $w \to \infty$ when $g(x)$ is a real valued C^1-smooth function on the whole real line in $L^1(\mathbb{R})$, meaning that $\int_{-\infty}^{\infty} |g(x)|\, dx$ is finite. Show that the same is true for $g \in L^1(\mathbb{R})$ that are merely continuous. Hint: You can use Weierstraß' theorem about uniformly approximating continuous functions on a closed interval by polynomials to deal with the part of the integral on $[-N, N]$ once N has been chosen.

5. Show that the Fourier transform of a continuous function in $L^1(\mathbb{R})$ tends to zero at $\pm\infty$. Hint: Observe that problem 4 is true with cos in place of sin and so ...