Lesson 40 36, 37, 38 due tonight. 39, 40, 41 Wed. next (42 never due)

Recording of WebEx Off. Hr. on Home Page next to Homework 11 assignment.

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> plot3d(exp(x)*cos(y), x=-2..2, y=0..6*Pi);
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p. 556: 19. Separation of variables Prob. (Not D'Alembert's)

12.10 Laplacian in polar coord.

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

p. 591: 4abc.

a) Apply Δ in polar coords to

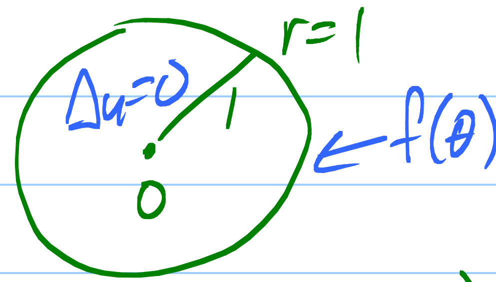
$r^n \cos n\theta$ and $r^n \sin n\theta$.

Show their Δ is $= 0$.

b) (*) $$u(r,\theta) = a_0 + \sum_{n=1}^{\infty} \left(a_n r^n \cos n\theta + b_n r^n \sin n\theta\right)$$

also satisfies $\Delta u = 0$ on disc

(if it is justified to put derivatives under Σ).



c) Aha!

$$u(1,\theta) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\theta + b_n \sin \theta\right)$$

is just a Fourier series. If I want to solve the Dirichet Prob. on the unit disc,

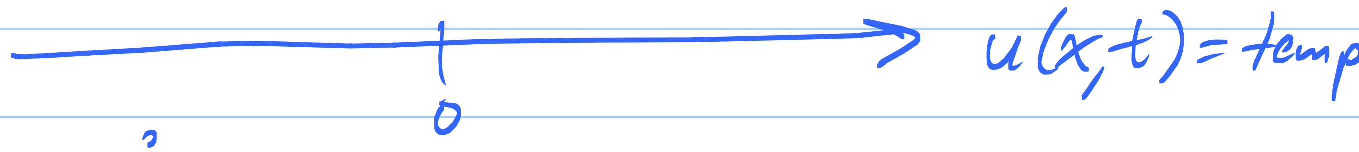
$\Delta u = 0$ inside, $u(1,\theta) = f(\theta)$ given

$(f(0) = f(2\pi))$, then (*) does it.

Lesson 41, 12.7 Heat problems on long wires.

Using Fourier Transform, $\hat{f}$, $\tilde{f}_c$, $\tilde{f}_s$ to solve heat problems (insulated end, iced end, no end). See Lessons 31, 32.

Hot infinite wire, given initial temp.



$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ plus Initial Cond, $u(x,0) = f(x)$.

New approach: Separate variables. Look for sol^{n}s $u(x,t) = X(x)T(t)$. Get

$$\frac{X''}{X} = \frac{T'}{c^2 T} = \lambda, \text{ const.}$$

No conditions on λ! Get many more sol^{n}s than when we had BCs.

Look at wiggling sol^{n}s. Let $\lambda = -w^2$. ($\lambda < 0$).

$X'' + w^2 X = 0 \qquad T' = -c^2 w^2 T$

$X(x) = A\cos wx + B\sin wx \qquad T = K e^{-c^2 w^2 t}$

Get sol^n $u(x,t) = (A\cos wx + B\sin wx)e^{-c^2 w^2 t}$

Big idea. No restriction on w.

Hmmm. Riemann sum is a linear combo of sol^{n}s.

$$\sum_{n=1}^{N}\Big(\underbrace{A(w_j)}_{\text{coeff}}\cos w_j x + \underbrace{B(w_j)}_{\text{coeff}}\sin w_j x\Big)e^{-c^2 w_j^2 t}\,\Delta w_j$$

$$\downarrow$$

(*) $$u(x,t) = \int_0^\infty \big(A(w)\cos wx + B(w)\sin wx\big)e^{-c^2 w^2 t}\,dw$$

If all goes well, this will solve PDE and we can diff under $\int$ to see that. Can solve

IC $u(x,0) \overset{\text{want}}{=} f(x) = \int_0^\infty A(w)\cos wx + B(x)\sin wx \, dw$

Aha! This is just the Fourier Integral for f:

$$A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\cos wv \, dv$$

$$B(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\sin wv \, dv$$

p. 574: 2-5. Just find $A(w)$ and $B(w)$ for given f and plug it into (*)

2. $f(x) = 1$ if $-a < x < a$ and 0 otherwise.

$$A(w) = \frac{1}{\pi}\int_{-a}^{a} 1 \cos wv \, dv = \frac{1}{\pi}\left[\frac{1}{w}\sin wv\right]_{v=-a}^{a}$$

$$B(w) = \frac{1}{\pi}\int_{-a}^{a} \underbrace{\underset{\text{even}}{1} \cdot \underset{\text{odd}}{\sin wv}}_{\text{odd}} \, dv = 0$$

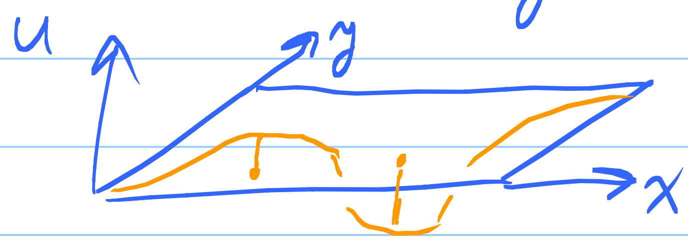
over a symmetric interval about 0

<u>Remark</u>: Can do calculus to clean up (*):

$$u(x,t) = \frac{1}{2c\sqrt{\pi t}}\int_{-\infty}^{\infty} f(v)\exp\left(\frac{-(x-v)^2}{4c^2 t}\right) dv$$

$$u(x,t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x + 2cz\sqrt{t})\, e^{-z^2}\, dz$$

Lesson 42 12.9 Vibrating square membrane.



$u(x,y,t)$ = displacement of drum at (x,y) at time t

Physics: $\dfrac{\partial^2 u}{\partial t^2} = c^2 \Delta u$ PDE

BC $u = 0$ on edges
IC initial shape and speed of drum head.

Separate variables: $u(x,y,t) = X(x)\,Y(y)\,T(t)$