Quiz 5 — MA261 — July 11, 2017

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1. Evaluate $\iint_R \sin\left(x^2+y^2\right)\,dA$, where R is the region in the first quadrant between the circles with center the origin and radii 1 and 3.

Substitute using $x = r\cos\theta$, $y = r\sin\theta$, and $dA = rdrd\theta$. Then the integral becomes

$$\begin{aligned}\int_0^{\pi/2}\int_1^3 \sin\left(r^2\right) r\;drd\theta &= \int_0^{\pi/2} d\theta \int_1^3 r\sin\left(r^2\right)dr \\ &= \frac{\pi}{2}\left[-\frac{1}{2}\cos\left(r^2\right)\right]_1^3 \\ &= \frac{\pi}{4}(\cos 1 - \cos 9)\end{aligned}$$

2. Evaluate $\int_0^2\int_0^3\int_0^z (2x-y)\;dxdydz$.

$$\begin{aligned}\int_0^2\int_0^3\int_0^z (2x-y)\;dxdydz &= \int_0^2\int_0^3 (x^2-yx)\Big|_0^z\,dydz \\ &= \int_0^2\int_0^3 (z^2-yz)\;dydz \\ &= \int_0^2 (yz^2-\frac{1}{2}y^2z)\Big|_0^3\,dz \\ &= \int_0^2 (3z^2-\frac{9}{2}z)\;dz \\ &= z^3-\frac{9}{4}z^2\Big|_0^2 \\ &= -1\end{aligned}$$