~~Fall 2011~~
Spring 2015

MA 16100
Study Guide - Exam # 1

1 Review of Algebra/PreCalculus:

(a) Distance between $P(x_1, y_1)$ and $P(x_2, y_2)$ is $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(b) Equations of lines:

(i) Point-Slope Form: $y - y_1 = m(x - x_1)$

(ii) Slope-Intercept Form: $y = mx + b$

(c) $L_1 || L_2 \iff m_1 = m_2$; $L_1 \perp L_2 \iff m_2 = -\dfrac{1}{m_1}$

(d) Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.

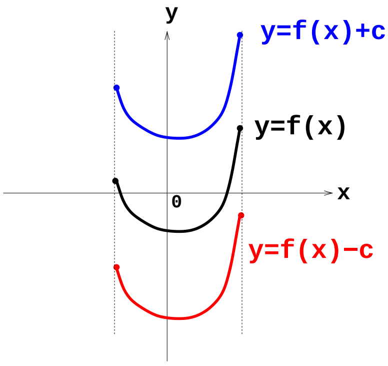
(e) Determining domain of a function $f(x)$.

2 Transformations of Functions $y = f(x)$

(I) Vertical Shift $(c > 0)$

(a) $y = f(x) + c \implies$ *shift* $f(x)$ vertically c units *up*.

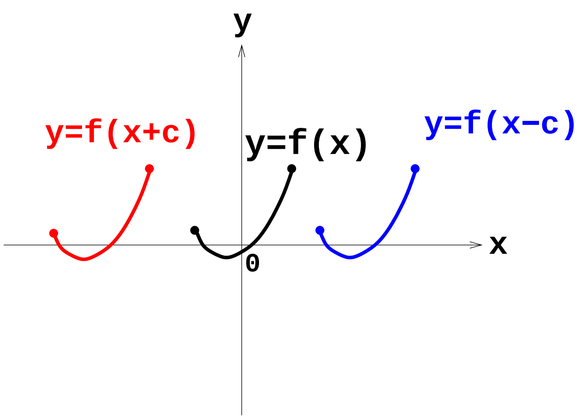
(b) $y = f(x) - c \implies$ *shift* $f(x)$ vertically c units *down*.



(II) Horizontal Shift $(c > 0)$

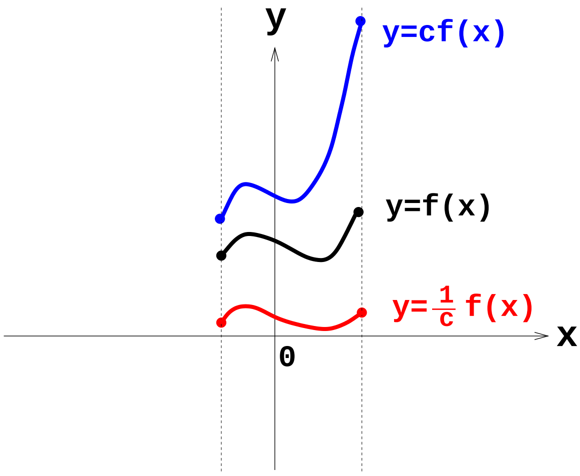
(a) $y = f(x - c) \implies$ *shift* $f(x)$ horizontally c units *right*.

(b) $y = f(x + c) \implies$ *shift* $f(x)$ horizontally c units *left*.



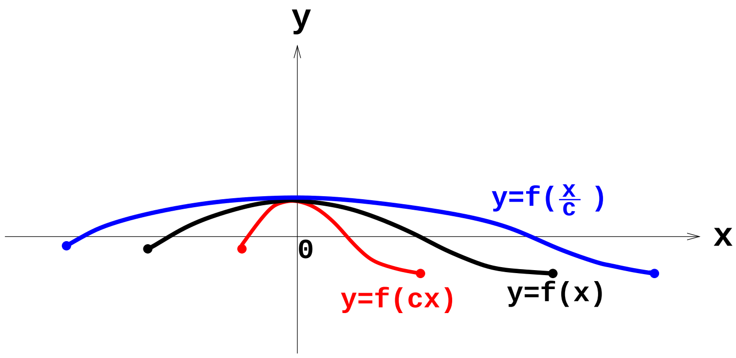
(III) Vertical Stretch/Shrink $(c > 0)$

$y = cf(x) \implies$ *stretch* $f(x)$ vertically by a factor c. (If $c < 1$, this shrinks the graph.)



(IV) Horizontal Stretch/Shrink $(c > 0)$

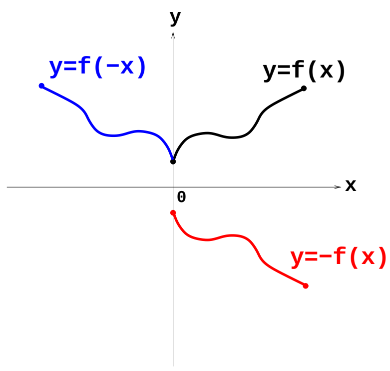
$y = f\left(\frac{x}{c}\right) \implies$ *stretch* $f(x)$ horizontally by a factor c. (If $c < 1$, this shrinks graph.)



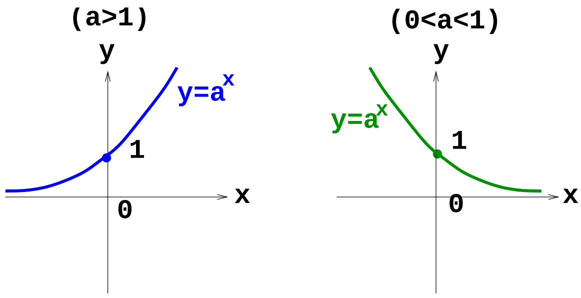
(V) Reflections

(a) $y = -f(x) \implies$ *reflect* $f(x)$ about x-axis

(b) $y = f(-x) \implies$ *reflect* $f(x)$ about y-axis



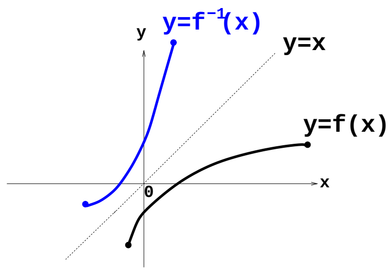
3 Combinations of functions; composite function $(f \circ g)(x) = f(g(x))$; $y = e^x$; exponential functions $y = a^x$ ($a > 0$ fixed):



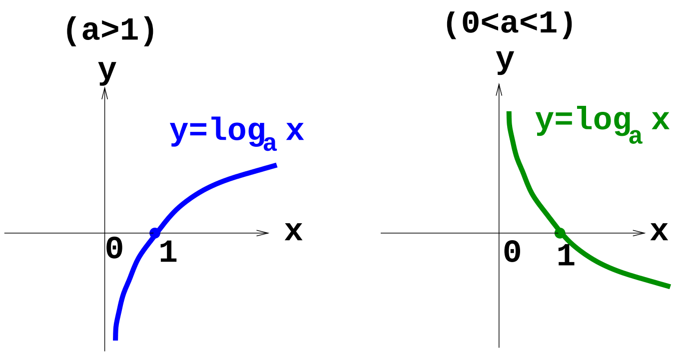
4 Law of Exponents:

$$a^{x+y} = a^x\, a^y$$
$$a^{x-y} = \frac{a^x}{a^y}$$
$$(a^x)^y = a^{xy}$$
$$(ab)^x = a^x\, b^x$$

5 One-to-one functions; `Horizontal Line Test`; inverse functions; finding the inverse $f^{-1}(x)$ of a 1-1 function $f(x)$; graphing inverse functions:



6 Logarithmic functions to base a: $y = \log_a x$ ($a > 0$, $a \neq 1$):



7 Logarithm formulas:

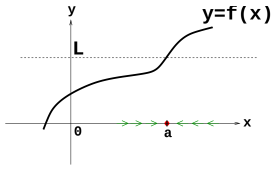
$$\log_a x = y \iff a^y = x$$
$$\log_a(a^x) = x, \text{ for every } x \in \mathbb{R}$$
$$a^{\log_a x} = x, \text{ for every } x > 0$$

8 Law of Logarithms:

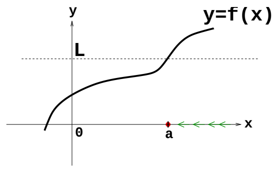
$$\log_a(xy) = \log_a x + \log_a y$$
$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a(x^p) = p\log_a x$$

9 Finite Limits

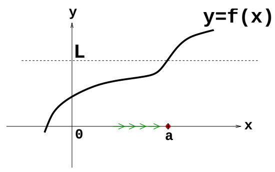
(a) $\lim_{x\to a} f(x) = L$



(b) $\lim_{x\to a^+} f(x) = L$ (right-hand limit)



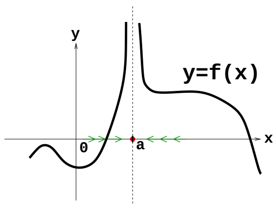
(c) $\lim_{x\to a^-} f(x) = L$ (left-hand limit)



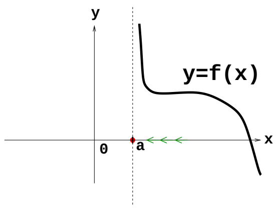
Recall: $\boxed{\lim_{x\to a} f(x) = L \iff \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L}$

10 Infinite Limits

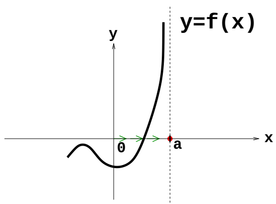
(a) $\lim_{x\to a} f(x) = \infty$



(b) $\lim_{x\to a^+} f(x) = \infty$



(c) $\lim_{x\to a^-} f(x) = \infty$



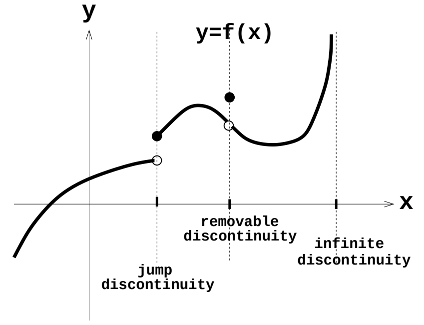
Remark: The line $x = a$ is a *Vertical Asymptote* of $f(x)$ if at least one of $\lim_{x\to a} f(x)$ or $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^-} f(x)$ is ∞ or $-\infty$.

11 `Limit Laws`; computing limits using Limit Laws;

<u>SQUEEZE THEOREM</u> : If $h_1(x) \le f(x) \le h_2(x)$ and $\lim_{x\to a} h_1(x) = \lim_{x\to a} h_2(x) = L$,

then $\lim_{x\to a} f(x) = L$

12 f continuous at a (i.e. $\lim_{x\to a} f(x) = f(a)$); f continuous on an interval; f continuous from the left at a (i.e. $\lim_{x\to a^-} f(x) = f(a)$) or continuous from the right at a (i.e. $\lim_{x\to a^+} f(x) = f(a)$); jump discontinuity, removable discontinuity, infinite discontinuity:



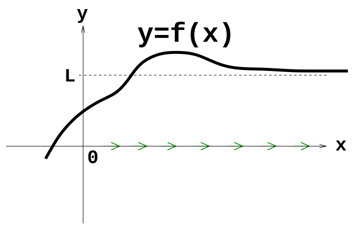
<u>LIMIT COMPOSITION THEOREM</u>: If f is continuous at b, where $\lim_{x\to a} g(x) = b$,

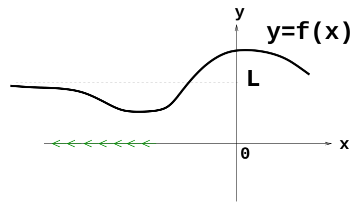
then $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right) = f(b)$.

13 **<u>Limits at Infinity</u>**

(a) $\lim_{x\to\infty} f(x) = L$

(b) $\lim_{x\to-\infty} f(x) = L$





<u>Remark</u>: The line $y = L$ is a *Horizontal Asymptote* of $f(x)$.

14 Average rate of change of $y = f(x)$ over the interval $[x_1, x_2]$: $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$;
slope of secant line through two points; average velocity. Definition of derivative of $y = f(x)$ at a: $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ or, equivalently, $f'(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$; interpretation of derivative:

$$f'(a) = \begin{cases} \text{slope of tangent line the graph of } y = f(x) \textit{ at } a \\ \text{velocity at time } a \\ \text{(instantaneous) rate of change of } f \text{ at } a \end{cases}$$

15 Derivative as a function: $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \dfrac{dy}{dx}$; differentiable functions (i.e., $f'(x)$ exists); higher order derivatives : $f''(x) = \dfrac{d^2y}{dx^2}, \ldots$.