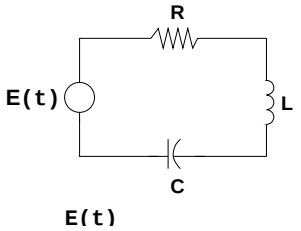
RLC-Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: ode45, plot routines.

Description: If $Q(t)$ = charge on a capacitor at time t in an RLC circuit (with R, L and C being the resistance, inductance and capacitance, respectively, and $E(t)$ = applied voltage), then Kirchoff's Laws gives the following 2^{nd} order differential equation for $Q(t)$:

$$(*) \qquad L\,Q''(t) + R\,Q'(t) + \frac{1}{C}\,Q(t) = E(t)$$



E(t)

Questions: Assume $L = 1$, $C = \frac{1}{4}$, $R = 2$ and $E(t) = 10\cos wt$.

(1) Use **ode45** to plot the solution of $(*)$ with $Q(0) = Q'(0) = 0$ over the interval $0 \le t \le 30$, for these values of w : $w = 1.0,\ 2.0,\ 4.0,\ 10.0,\ 20.0$.

(2) For each of these 5 plots, find the largest value of $|Q(t)|$ over $10 \le t \le 30$ and fill in the table:

w	Max value of $\lvert Q(t)\rvert$ on $10 \le t \le 30$
1.0	
2.0	
4.0	
10.0	
20.0	

(3) Does increasing w appear to increase the maximum charge $|Q(t)|$ on the capacitor ?

Remark: There is an analogy between spring-mass systems and RLC circuits given by :

Spring-Mass System	RLC Circuit
$\boldsymbol{mx'' + cx' + kx = F(t)}$	$\boldsymbol{LQ'' + RQ' + \frac{1}{C}\,Q = E(t)}$
$x =$ Displacement	$Q =$ Charge
$x' =$ Velocity	$Q' = I =$ Current
$m =$ Mass	$L =$ Inductance
$c =$ Damping constant	$R =$ Resistance
$k =$ Spring constant	$\frac{1}{C} =$ (Capacitance)$^{-1}$
$F(t) =$ External force	$E(t) =$ Voltage