a. $n = 0$: $\alpha = 0$

b. $n = 1$: $\alpha = 0$ and $\alpha = 1$

c. $n = 2$: $\alpha = 0$ and $\alpha = 2$

5. Formula (9) for $f''(x)$ is often used in the numerical solution of differential equations. By adding the Taylor series for $f(x + h)$ and for $f(x - h)$, show that the error in this formula has the form $\sum_{n=1}^{\infty} a_{2n} h^{2n}$. Determine the coefficients a_{2n} explicitly. Also derive the error term given in Equation (9).

6. Derive the following two formulas for approximating derivatives and show that they are both $\mathcal{O}(h^4)$ by establishing their error terms:

$$f'(x) \approx \frac{1}{12h}[-f(x + 2h) + 8f(x + h) - 8f(x - h) + f(x - 2h)]$$

$$f''(x) \approx \frac{1}{12h^2}[-f(x + 2h) + 16f(x + h) - 30f(x) + 16f(x - h) - f(x - 2h)]$$

7. Derive the following two formulas for approximating the third derivative. Find their error terms. Which formula is more accurate?

$$f'''(x) \approx \frac{1}{h^3}[f(x + 3h) - 3f(x + 2h) + 3f(x + h) - f(x)]$$

$$f'''(x) \approx \frac{1}{2h^3}[f(x + 2h) - 2f(x + h) + 2f(x - h) - f(x - 2h)]$$

8. (Continuation) Carry out the instructions in the preceding problem for the following fourth-derivative formulas:

$$f^{(4)} \approx \frac{1}{h^4}[f(x + 4h) - 4f(x + 3h) + 6f(x + 2h) - 4f(x + h) + f(x)]$$

$$f^{(4)} \approx \frac{1}{h^4}[f(x + 2h) - 4f(x + h) + 6f(x) - 4f(x - h) + f(x - 2h)]$$

9. Show that in Richardson extrapolation,

$$D(2, 2) = \frac{16}{15}\psi(h/2) - \frac{1}{15}\psi(h)$$

10. Show how to use Richardson extrapolation employing x_n and x_{n^2} if

$$L = x_n + a_1 n^{-1} + a_2 n^{-2} + a_3 n^{-3} + \cdots$$

11. Prove or disprove:

a. If $L - x_n = \mathcal{O}(n^{-1})$, then $L - (2x_{2n} - x_n) = \mathcal{O}(n^{-2})$.

b. If $L - x_n = \mathcal{O}(n^{-1})$, then $L - x_{n^2} = \mathcal{O}(n^{-2})$.

Discuss the numerical consequences of this problem.

12. Show how to use Richardson extrapolation if

$$L = \varphi(h) + a_1 h + a_3 h^3 + a_5 h^5 + \cdots$$

13. Suppose that $L = \lim_{h \to 0} f(h)$ and that $L - f(h) = c_6 h^6 + c_9 h^9 + \cdots$. What combination of $f(h)$ and $f(h/2)$ should be the best estimate of L?

14. Using Taylor series, derive the error term for the approximation

$$f'(x) \approx \frac{1}{2h}[-3f(x) + 4f(x + h) - f(x + 2h)]$$