7.1

7. By using the Taylor series of $f(x+kh)$, $f(x+3h)-3f(x+2h)+3f(x+h)-f(x)=h^3f^{'''}(x)+O(h^4)$ Hence, $f^{'''}(x)=\frac{1}{h^3}(f(x+3h)-3f(x+2h)+3f(x+h)-f(x))+O(h)$. Similarly we can compute the second approximation, $f^{'''}(x)=\frac{1}{2h^3}(f(x+2h)-2f(x+h)+2f(x-h)-f(x-2h))+O(h^2)$. Hence the second approximation is more accurate.

10. Replacing n by n^2, $L=x_{n^2}+a_1n^{-2}+a_2n^{-4}+a_3n^{-6}+\cdots$. Mutiflying n and subtract this from given equation, $(1-n)L=x_n-nx_{n^2}+O(n^{-2})$. Hence $L=\frac{n}{n-1}x_{n^2}-\frac{1}{n-1}x_n+O(n^{-3})$

12. Replacing h by $\frac{h}{2}$, $L=\phi(\frac{h}{2})+a_1\left(\frac{h}{2}\right)+a_3\left(\frac{h}{2}\right)^3+\cdots$. Multiflying by 2 and subtract given equation from this, $L=2\phi(\frac{h}{2})-\phi(h)+O(h^3)$. we can do same computiaon on page 472-476.

13. $L=f(h)+c_6h^6+c_9h^9+\cdots$. Replacing h by $\frac{h}{2}$, $L=f(\frac{h}{2})+\frac{c_6}{2^6}h^6+\frac{c_9}{2^9}h^9+\cdots$. Multiflying 2^6 and subtract the first equation from this, $(2^6-1)L=2^6f(\frac{h}{2})-f(h)+(\frac{1}{2^3}-1)c_9+\cdots=64f(\frac{h}{2})-f(h)-\frac{7}{8}c_9+\cdots$. Hence $L=\frac{64f(\frac{h}{2})}{63}-\frac{f(h)}{63}-\frac{1}{72}c_9+\cdots=\frac{64}{63}f(\frac{h}{2})-\frac{1}{63}f(h)+O(h^9)$. So the best combination of $f(h)$ and $f(\frac{h}{2})$ as a estimate of L is $\frac{64}{63}f(\frac{h}{2})-\frac{1}{63}f(h)$