7.2

4. All polynomials of degree ≤ 4 are linear combination of $1, x, x^2, x^3$ and x^4. Hence it suffies to show that the given eqauiton holds for them. For example, let $f(x) = 1$. $\int_0^1 f(x)dx = \int_0^1 1dx = 1$ and $\frac{1}{90}\left(7f(0) + 32f(\frac{1}{4})12f(\frac{1}{2}) + 32f(\frac{3}{4}) + 7f(1)\right) = 1$.

8. Just plug $f(x)$ or, e^x and $\cos(\frac{x\pi}{2})$, in the given eqaution. Then you can find $A_0 = \frac{2}{\pi}$ and $A_1 = \frac{1}{e}(e - 1 - \frac{2}{\pi})$.

10. $l_0(x) = -3(x - \frac{2}{3})$ and $l_1(x) = 3(x - \frac{1}{3})$. Plug them in the given eqution. Then $A = B = \frac{1}{2}$. Let $x = \lambda(t) = (b-a)t + a$. Hence $\int_a^b f(x)dx = \int_{\lambda(0)}^{\lambda(1)} f(\lambda(t))(b-a)dt = \frac{(b-a)}{2}[f(\lambda(\frac{1}{3})) + f(\lambda(\frac{2}{3}))] = \frac{(b-a)}{2}(f(\frac{(2a+b)}{3}) + f(\frac{(a+2b)}{3}))$.

19. $\int_a^b f(x)dx = \sum_{i=1,odd}^{n} \int_{x_{i-1}}^{x_{i+1}} f(x)dx \approx \sum_{i=1,odd}^{n}(x_{i+1} - x_{i-1})f(x_i) = 2h\sum_{i=1}^{\frac{n}{2}} f(x_{2i-1})$.