2.2

8. $x = \frac{-b+\sqrt{b^2-4ac}}{2a}\frac{(-b+\sqrt{b^2-4ac})}{(-b+\sqrt{b^2-4ac})} = \frac{-2c}{-b+\sqrt{b^2-4ac}}$ works if $b > 0$ and $x = \frac{-b-\sqrt{b^2-4ac}}{2a}\frac{(-b-\sqrt{b^2-4ac})}{(-b-\sqrt{b^2-4ac})} = \frac{-2c}{-b-\sqrt{b^2-4ac}}$ works if $b < 0$.

9.(a)$\sqrt{x^2+1} - x = (\sqrt{x^2+1} - x)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} = \frac{1}{\sqrt{x^2+1}+x}$.

(b)$\log x - \log y = \log \frac{x}{y}$

(e)Using the Taylor series for e^x, $e^x - e = \sum_{k=0}^{\infty} \frac{x^k}{k!} - e$

(f)$\log x - 1 = \log x - \log 10 = \log \frac{x}{10}$

(h)$\sin x - \tan x = \sin x - \frac{\sin x}{\cos x} = \frac{\sin x \cos x - \sin x}{\cos x} = \frac{\sin x(\cos x - 1)}{\cos x} = \frac{\sin x}{\cos x}(\cos x - 1)\frac{\cos x+1}{\cos x+1} = \frac{(\tan x)(-\sin^2 x)}{\cos x+1}$

15.(a) To make $f(x)$ continuous at $x = 0$, $f(0) = \lim_{x\to 0} f(x)$. Hence $f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{\sin x}{1} = 0$, by LHospital's Rule.

(b)$x = 2k\pi, \quad k \in \mathbb{Z}$. (since $\cos 2k\pi \approx 1$)

(c)$f(x) = \frac{1-\cos x}{x} = \frac{1-\cos x}{x}\frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{x(1+\cos x)} = \frac{\sin^2 x}{x(1+\cos x)}$

(d)With the formula in (c), we have diffuculty when $\cos x \approx -1$. We can use original equation to avoid this difficulty.

2.3

7. Characteristic Equation: $x^2 - 2x - 1 = 0$. Roots are $x = 1 \pm \sqrt{2}$. Hence a general solution is $x_n = A(1+\sqrt{2})^n + B(1-\sqrt{2})^n$. This recurrence relation is not a good way to compute x_n since $1+\sqrt{2} > 1$.

8. Characteristic Equation: $x^2 - x - 1 = 0$. Roots are $x = \frac{1\pm\sqrt{5}}{2}$. Using initial conditions, then $r_n = \frac{(1+\sqrt{5})^{n+1}-(1-\sqrt{5})^{n+1}}{\sqrt{5}2^{n+1}}$. Hence,

$$\begin{aligned}\frac{2r_n}{r_{n-1}} &= \frac{2[(1+\sqrt{5})^{n+1}-(1-\sqrt{5})^{n+1}]}{\sqrt{5}2^{n+1}}\frac{\sqrt{5}2^n}{(1+\sqrt{5})^n-(1-\sqrt{5})^n]} = \frac{(1+\sqrt{5})^{n+1}-(1-\sqrt{5})^{n+1}}{(1+\sqrt{5})^n-(1-\sqrt{5})^n} \\ &= \frac{(1+\sqrt{5})-(1-\sqrt{5})\left(\frac{1-\sqrt{5}}{1+\sqrt{5}}\right)^n}{1-\left(\frac{1-\sqrt{5}}{1+\sqrt{5}}\right)^n} \to 1+\sqrt{5} \quad \textit{as} \quad n\to\infty \quad \textit{since} \quad 0 < \frac{1-\sqrt{5}}{1+\sqrt{5}} < 1.\end{aligned}$$

9. Since $\frac{1+\sqrt{5}}{2} > 1$, the recurrence relation doesn't provide a stable means for computing r_n. Actually, with given inital conditions, $r_n = \left(\frac{1-\sqrt{5}}{2}\right)^{n-1}$. And this goes to 0 as n goes to infinity.