3.1

4. By Theorem 1, $|r-c_n| \le 2^{-(n+1)}(b_0-a_0) \le \epsilon$

Hence, $n \ge \dfrac{\log(b_0-a_0)-\log\epsilon}{\log 2} - 1$

5. By Theorem 1, $\dfrac{|r-c_n|}{r} \le \dfrac{|r-c_n|}{a_0} \le \dfrac{2^{-(n+1)}(b_0-a_0)}{a_0} \le \epsilon$

Hence, $n \ge \dfrac{\log(b_0-a_0)-\log\epsilon-\log a_0}{\log 2} - 1$

7. (1) Absolute Accuracy

By Problem 4, $n \ge \dfrac{\log(b_0-a_0)-\log\epsilon}{\log 2} - 1 = \dfrac{0+6}{\log 2} - 1 = 18.93$. For Marc-32, $\epsilon = 2^-24$. Hence, $n \ge \dfrac{0+24\log 2}{\log 2} - 1 = 23$.

(2) Relative Accuracy

By Problem 5, $n \ge \dfrac{\log(b_0-a_0)-\log\epsilon-\log a_0}{\log 2} - 1 = \dfrac{0-\log 2+6}{\log 2} - 1 = 17.93$. For Marc-32, $\epsilon = 2^-24$. Hence, $n \ge \dfrac{0+24\log 2-\log 2}{\log 2} - 1 = 22$.

3.2

5. $x_{n+1} = 2x_n - x_n^2 y = x_n - (x_n^2 y - x_n) = x_n - \dfrac{y-\frac{1}{x_n}}{\frac{1}{x_n^2}}$. Hence $f(x) = y - \dfrac{1}{x}$

10. Let $f(x) = x^3 - R$. Then $f'(x) = 3x^2$ and $f''(x) = 6x$. Hence, if $x > 0$, by Theorem 2, we can find a zero from any initial point. If $x < 0$, $x_{n+1} = x_n - \dfrac{f(x)}{f'(x)} = x_n - \dfrac{x_n^3 - R}{3x_n^2} = \dfrac{2x_n^3+R}{3x_n^2}$. Since x_{n+1} can be zero, Newton's Method can diverge. For example, if we choose x_0 such taht $2x_0^3 + R = 0$,(i.e $x_0 = \left(-\dfrac{2}{R}\right)^{\frac{1}{3}}$, then $x_1 = 0$

15. $0 = f(r) = f(x_n - e_n) = f(x_n) - e_n f'(\xi_n) \Rightarrow f(x_n) = e_n f'(\xi_n)$. Hence $e_{n+1} = x_{n+1} - r = x_n - \dfrac{f(x_n)}{f'(x_0)} - r = x_n - r - \dfrac{f(x_n)}{f'(x_0)} = e_n - \dfrac{e_n f'(\xi_n)}{f'(x_0)} = \left(1 - \dfrac{f'(\xi_n)}{f'(x_0)}\right) e_n$. Hence $C = 1 - \dfrac{f'(\xi_n)}{f'(x_0)}$ and $s = 1$.

17. (b). Let $a_n = \dfrac{1}{2^{2^n}}$.

Then, $a_{n+1} = \dfrac{1}{2^{2^{n+1}}} = \dfrac{1}{2^{2^n\cdot 2}} = \dfrac{1}{(2^{2^n})^2} = \left(\dfrac{1}{2^{2^n}}\right)^2 = a_n^2$.

19. $0 = f(r) = f'(r) = \cdots = f^{l-1}(r)$ and $f^l(r) \ne 0$ where $k \le l$.

Hence $f(x_n) = f(r+e_n) = f(r) + f'(r)e_n + \cdots + \dfrac{f^{l-1}(r)}{(l-1)!}e_n^{l-1} + \dfrac{f^l(r)}{l!}e_n^l + \dfrac{f^{l+1}(\xi)}{(l+1)!}e_n^{l+1} = \dfrac{f^l(r)}{l!}e_n^l + \dfrac{f^{l+1}(\xi)}{(l+1)!}e_n^{l+1}$.

$f'(x_n) = f'(r+e_n) = f'(r) + f''(r)e_n + \cdots + \dfrac{f^l(r)}{(l-1)!}e_n^{l-1} + \dfrac{f^{l+1}(\nu)}{l!}e_n^l = \dfrac{f^l(r)}{(l-1)!}e_n^{l-1} + \dfrac{f^{l+1}(\nu)}{l!}e_n^l$.

Then $e_{n+1} = x_{n+1} - r = x_n - \dfrac{kf(x_n)}{f'(*x_n)} - r = e_n - \dfrac{kf(x_n)}{f'(*x_n)}$. Plugginh first 2 equations in the last. Then we have, $e_{n+1} = e_n^2 \cdot A$ where A is an equation which can be bounded.

21. Let $g(x) = \dfrac{f(x)}{\sqrt{f'(x)}}$. then $g'(x) = \dfrac{2(f'(x))^2 - f(x)f''(x)}{2f'(x)\sqrt{f'(x)}}$.
$x_{n+1} - \dfrac{g(x)}{g'(x)} = x_n - \dfrac{2f'(x_n)f(x_n)}{2f'(x_n)^2 - f(x_n)f''(x_n)}$.

23(a). $f_1(x_1, x_2) = 4x_1^2 - x_2^2$, $f_2(x_1, x_2) = 4x_1x_2^2 - x_1 - 1$ and $J = \begin{bmatrix} 8x_1 & -2x_2 \\ 4x_2^2 - 1 & 8x_1x_2 \end{bmatrix}$.
$J\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 3 & 0 \end{bmatrix}$ and $\left(J\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)^{-1} = \dfrac{1}{6}\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$. $f_1(0,1) = -1$ and $f_2(0,1) = -1$.
$\begin{bmatrix} h_1^0 \\ h_2^0 \end{bmatrix} = -J^{-1}\begin{bmatrix} f_1(0,1) \\ f_2(0,1) \end{bmatrix} = -\dfrac{1}{6}\begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{2} \end{bmatrix}$, $\begin{bmatrix} x_1^1 \\ x_2^1 \end{bmatrix} = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix} + \begin{bmatrix} h_1^0 \\ h_2^0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix}$.
$J\begin{bmatrix} \frac{1}{3} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{8}{3} & -1 \\ 0 & \frac{4}{3} \end{bmatrix}$ and $\left(J\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)^{-1} = \dfrac{9}{32}\begin{bmatrix} \frac{4}{3} & 1 \\ 0 & \frac{8}{3} \end{bmatrix}$. $f_1(\dfrac{1}{3}, \dfrac{1}{2}) = \dfrac{7}{36}$ and $f_2(\dfrac{1}{3}, \dfrac{1}{2}) = -1$.
$\begin{bmatrix} h_1^1 \\ h_2^1 \end{bmatrix} = -\dfrac{9}{32}\begin{bmatrix} \frac{4}{3} & 1 \\ 0 & \frac{8}{3} \end{bmatrix}\begin{bmatrix} \frac{7}{36} \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{5}{24} \\ \frac{3}{4} \end{bmatrix}$. Hence, $\begin{bmatrix} x_1^2 \\ x_2^2 \end{bmatrix} = \begin{bmatrix} x_1^1 \\ x_2^1 \end{bmatrix} + \begin{bmatrix} h_1^1 \\ h_2^1 \end{bmatrix} = \begin{bmatrix} \frac{13}{24} \\ \frac{5}{4} \end{bmatrix}$.

3.3

3. By Taylor expansion, $f(x+h) = f(x) + f](x)h + O(h^2)$. Hence $f(x+h) \approx f(x) + f'(x)h \rightarrow f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$. Similarly $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$. Then we have,
$kf'(x) - hf'(x) \approx k\left(\dfrac{f(x+h) - f(x)}{h}\right) - h\left(\dfrac{f(x+k) - f(x)}{k}\right) = \dfrac{k^2 f(x+h) - h^2 f(x+k) + (h^2 - k^2)f(x)}{(k-h)kh}$

Computer Problem 3.2

5. 0.1213 and 0.1231

13. $x = -0.2932$ and $y = 1.1727$. If we plug $z = x + yi$ in the equation on Problem 11. Then we have the eqautions on this problem. So they have same numerical behavior.

Computer Problem 3.3

3(a). 1.3917452000271