6.3

1.

0	2	-9	3	7	5
0	2	-6	10	17	
1	-4	4	44		
1	-4	48			
2	44				

Hence $p(x) = 2 - 9x + 3x^2 + 7x^2(x-1) + 5x^2(x-1)^2$.

3. We can use the equation (9). Hence $p(x) = \sum_{i=0}^{n} y_i A_i(x)$

4. Let $p(x) = a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^3$. Then $p'(x) = b + 2c(x - x_0) + 3d(x - x_0)^2$ and $p''(x) = 2c + 6d(x - x_0)$. $p(x_0) = a = c_{00}$ and $p''(x) = 2c = c_{02}$. So we can determine a and c. Let $h = x_1 - x_0$. Then $p''(x_1) = 2c + 6dh = c_{12}$. So $dh = \frac{2c - c_{12}}{6}$. To get the solution for d, $h \neq 0$. With $h \neq 0$, we can also find b by $p(x_1)$. Hence we need a condition $x_0 \neq x_1$

6.4

5. $f(1^-) = f(1^+) = 1$, $f(2^-) = f(2^+) = \frac{3}{2}$, $f'(1^-) = f'(1^+) = 1$ and $f'(2^-) = f'(2^+) = 0$. Hence $f(x)$ is a quadratic spline function.

7. (1) Similar to #5, we just check the continuity for $f(x)$, $f'(x)$, and $f''(x)$. Then we need $a = c = d$. (2) $f(1) = a = 7 = c = d$, $f(0) = 4a - b = 26$, Hence $b = 2$. $f(4) = 4d + e = 25$ Hence, $e = -3$.

9.

$$Q(x) = \begin{cases} Q_0(x) & x \in [t_0, t_1] \\ Q_1(x) & x \in [t_1, t_2] \\ \vdots & \\ Q_{n-1}(x) & x \in [t_{n-1}, t_n] \end{cases}$$

Each $Q_i(x)$ is a quadratic, so we have $3n$ coefficients. There are $2n$ continuity conditions $Q_i(t_i) = y_i$ and $Q_i(t_{i+1}) = y_{i+1}$ for $i = 0, .., n-1$. And also there are $n-1$ conditions, $Q'_{i-1}(t_i) = Q'_i(t_i)$ for $i = 1, ..., n-1$. Hence there are $3n - 1$ equations for $3n$ unknows. So one degree of freedom remains. We use this by letting $z_0 = 0$. Let $h_i = t_{i+1} - t_i$ and $Q'_i(x) = z_i \frac{t_{i+1} - x}{h_i} + z_{i+1} \frac{x - t_i}{h_i}$. Integrating both sides then we get $Q_i(x) = \frac{z_i}{2h_i}(t_{i+1} - x)^2 + \frac{z_{i+1}}{2h_i}(x - t_i)^2 + C$. By $Q_i(t_i) = y_i$, we can determine $C = y_i - \frac{z_i h_i}{2}$. Hence, $Q_i(x) = \frac{z_i}{2h_i}(t_{i+1} - x)^2 + \frac{z_{i+1}}{2h_i}(x - t_i)^2 + y_i - \frac{z_i h_i}{2}$. By $Q_{i-1}(t_i) = y_i$, we get $y_i = \frac{1}{2} z_i h_{i-1} + y_{i-1} - \frac{1}{2} z_{i-1} h_{i-1}$. So $z_i = \frac{2(y_i - y_{i-1})}{t_i - t_{i-1}} + z_{i-1}$. Thus using $z_0 = 0$, we can determine all z_i 's.

13. $f(0) = 1$, $f(1^-) = f(1^+) = 1$, $f(2^-) = f(2^+) = 0$, and $f(3) = 10$. $f'(1^-) = f'(1^+)$, $f'(2^-) = f'(2^+)$, $f''(1^-) = f''(1^+)$, $f''(2^-) = f''(2^+)$, and $f''(0) = f''(3) = 0$. So $f(x)$ is the natural cubic spline for the given table.

24. $\int_0^1 S(x) = \sum_{i=0}^{n-1} \frac{(t_{i+1} - t_i)(f(t_i) + f(t_{i+1}))}{2}$

30. The first row of matrix system on page 352 must be replaced by $h_0 z_0 + u_1 z_1 + h_1 z_1 = v_1$ and the last row must be replaced by $h_{n-2} z_{n-2} + u_{n-1} z_{n-1} + h_{n-1} z_n = v_{n-1}$. Then we can use the equation (8) when $i = 0$ and the equation (9) when $i = n$ to find all the z_i for $i = 1, ..., n-1$.