6.5
7.

$$\int_{-\infty}^{\infty} B_i^k(x)dx = \lim_{x\to\infty}\int_{-\infty}^{x} B_i^k(x)dx = \lim_{x\to\infty}\left(\frac{t_{i+k+1}-t_i}{k+1}\right)\sum_{j=i}^{\infty} B_j^{k+1}(x) \quad (by \quad Lemma 7)$$

$$= \lim_{x\to\infty}\left(\frac{t_{i+k+1}-t_i}{k+1}\right)\sum_{j=-\infty}^{\infty} B_j^{k+1}(x) \quad (since\, for\, sufficently\, large\, x,\ \sum_{j=-\infty}^{i-1} B_j^{k+1}(x)=0)$$

$$= \lim_{x\to\infty}\left(\frac{t_{i+k+1}-t_i}{k+1}\right)(1) \quad (by \quad Lemma 4)$$

$$= \frac{t_{i+k+1}-t_i}{k+1}$$

6.7

6. We know that $e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!}$.

$$\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt = \frac{2}{\sqrt{\pi}}\int_0^x \sum_{k=0}^{\infty}\frac{(-t^2)^k}{k!}dt = \frac{2}{\sqrt{\pi}}\int_0^x \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}t^{2k}dt$$

$$= \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\frac{x^{2k+1}}{2k+1} \quad (by \quad Theorme 2)$$

14. We know that $\ln(1-x) = \sum_{k=1}^{\infty}\left(-\frac{x^k}{k}\right)$.

$$f(x) = -\int_0^x \frac{\ln(1-t)}{t}dt = -\int_0^x \frac{1}{t}\sum_{k=1}^{\infty}\left(-\frac{t^k}{k}\right)dt = \int_0^x \sum_{k=1}^{\infty}\frac{t^{k-1}}{k}dt = \sum_{k=1}^{\infty}\frac{x^k}{k^2} \quad (by \quad Theorme 2)$$

$\lim_{n\to\infty}\frac{|x^{n+1}/(n+1)^2|}{|x^n/n^2|} = \lim_{n\to\infty}|x|\frac{n^2}{(n+1)^2} = |x|$. By the ratio test, the radius of convergence is 1. Hence we can compute $f(0.001)$ by using this series. But $|-2| > 1$, so we can't use this series to compute $f(-2)$. Instead we can compute it by using Taylor series of $\frac{\ln(1-x)}{x}$ at $x = 2$