MA 16100
Study Guide - Final Exam

1 **Review of Algebra/PreCalculus**:

(a) Distance between $P(x_1, y_1)$ and $P(x_2, y_2)$ is $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

(b) Equations of lines:

  (i) Point-Slope Form: $y - y_1 = m(x - x_1)$

  (ii) Slope-Intercept Form: $y = mx + b$

(c) $L_1 || L_2 \iff m_1 = m_2$; $L_1 \perp L_2 \iff m_2 = -\dfrac{1}{m_1}$

(d) Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.

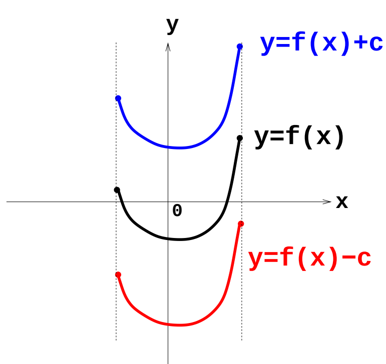
(e) Determining domain of a function $f(x)$.

2 **Transformations of Functions $y = f(x)$**

(I) Vertical Shift $(c > 0)$

(a) $y = f(x) + c \implies$ *shift* $f(x)$ vertically c units *up*.

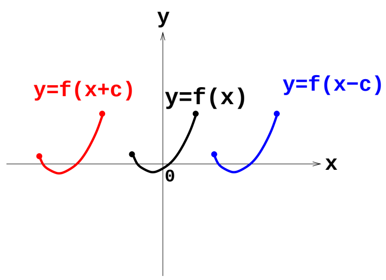
(b) $y = f(x) - c \implies$ *shift* $f(x)$ vertically c units *down*.



(II) Horizontal Shift $(c > 0)$

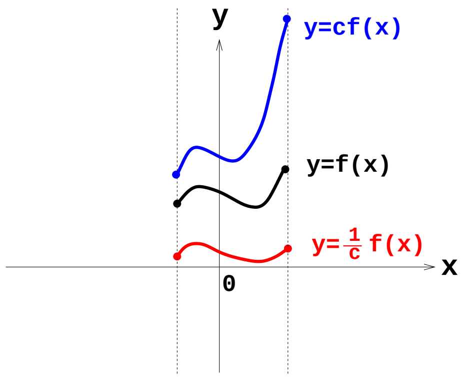
(a) $y = f(x - c) \implies$ *shift* $f(x)$ horizontally c units *right*.

(b) $y = f(x + c) \implies$ *shift* $f(x)$ horizontally c units *left*.



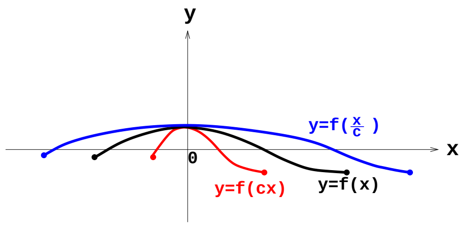
(III) <u>Vertical Stretch/Shrink</u> $(c > 0)$

$y = cf(x) \implies$ *stretch* $f(x)$ vertically by a factor c. (If $c < 1$, this shrinks the graph.)



(IV) <u>Horizontal Stretch/Shrink</u> $(c > 0)$

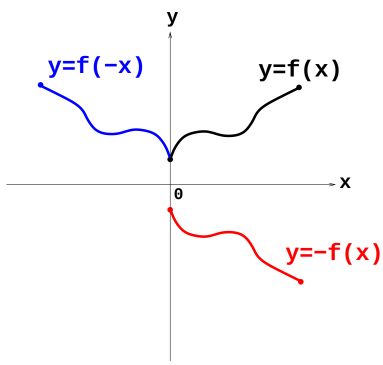
$y = f\left(\frac{x}{c}\right) \implies$ *stretch* $f(x)$ horizontally by a factor c. (If $c < 1$, this shrinks graph.)



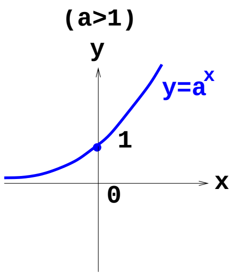
(V) <u>Reflections</u>

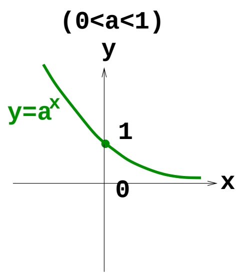
(a) $y = -f(x) \implies$ *reflect* $f(x)$ about x-axis

(b) $y = f(-x) \implies$ *reflect* $f(x)$ about y-axis



3 Combinations of functions; composite function $(f \circ g)(x) = f(g(x))$; $y = e^x$; exponential functions $y = a^x$ ($a > 0$ fixed):

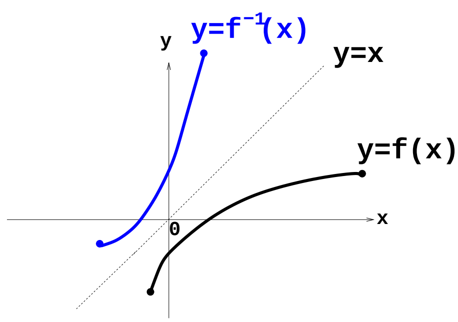




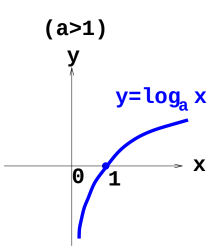
4 Law of Exponents:

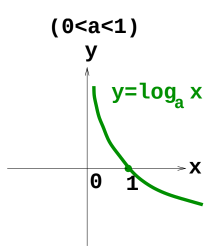
$$a^{x+y} = a^x\, a^y$$
$$a^{x-y} = \frac{a^x}{a^y}$$
$$(a^x)^y = a^{xy}$$
$$(ab)^x = a^x\, b^x$$

5 One-to-one functions; `Horizontal Line Test`; inverse functions; finding the inverse $f^{-1}(x)$ of a 1-1 function $f(x)$; graphing inverse functions:



6 Logarithmic functions to base a: $y = \log_a x$ ($a > 0$, $a \neq 1$):





7 Logarithm formulas:

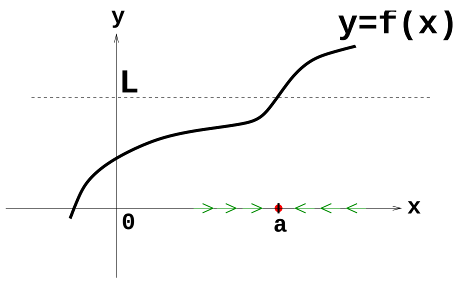
$$\log_a x = y \iff a^y = x$$
$$\log_a(a^x) = x, \text{ for every } x \in \mathbb{R}$$
$$a^{\log_a x} = x, \text{ for every } x > 0$$

8 Law of Logarithms:

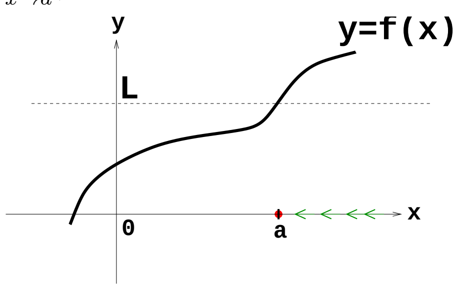
$$\log_a(xy) = \log_a x + \log_a y$$
$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a(x^p) = p\log_a x$$

9 **Finite Limits**

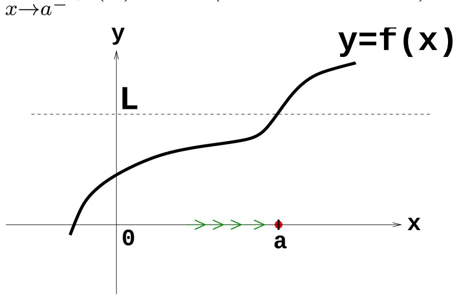
(a) $\lim_{x\to a} f(x) = L$



(b) $\lim_{x\to a^+} f(x) = L$ (right-hand limit)



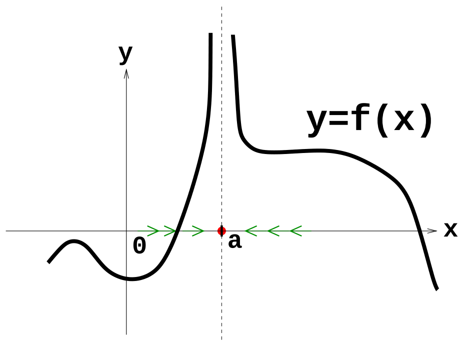
(c) $\lim_{x\to a^-} f(x) = L$ (left-hand limit)



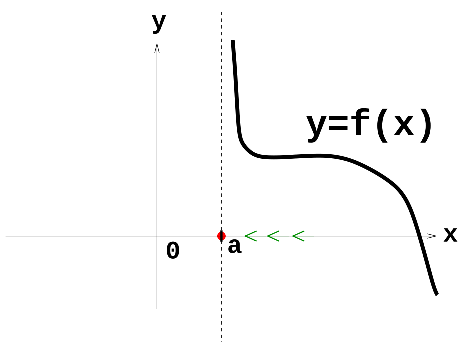
Recall: $\boxed{\lim_{x\to a} f(x) = L \iff \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L}$

10 Infinite Limits

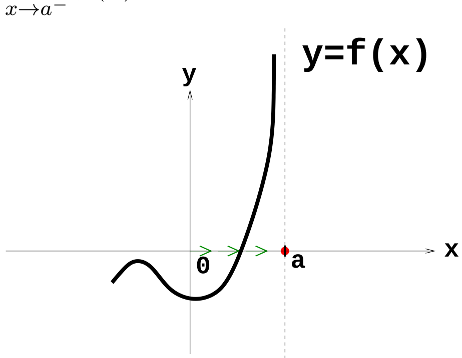
(a) $\lim_{x \to a} f(x) = \infty$



(b) $\lim_{x \to a^+} f(x) = \infty$



(c) $\lim_{x \to a^-} f(x) = \infty$



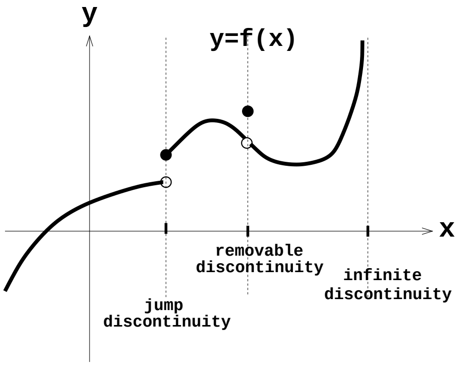
Remark: The line $x = a$ is a *Vertical Asymptote* of $f(x)$ if at least one of $\lim_{x \to a} f(x)$ or $\lim_{x \to a^+} f(x)$ or $\lim_{x \to a^-} f(x)$ is ∞ or $-\infty$.

11 `Limit Laws`; computing limits using Limit Laws;

SQUEEZE THEOREM : If $h_1(x) \le f(x) \le h_2(x)$ and $\lim_{x \to a} h_1(x) = \lim_{x \to a} h_2(x) = L$,

then $\lim_{x \to a} f(x) = L$

12 f continuous at a (i.e. $\lim_{x\to a} f(x) = f(a)$); f continuous on an interval; f continuous from the left at a (i.e. $\lim_{x\to a^-} f(x) = f(a)$) or continuous from the right at a (i.e. $\lim_{x\to a^+} f(x) = f(a)$); jump discontinuity, removable discontinuity, infinite discontinuity:

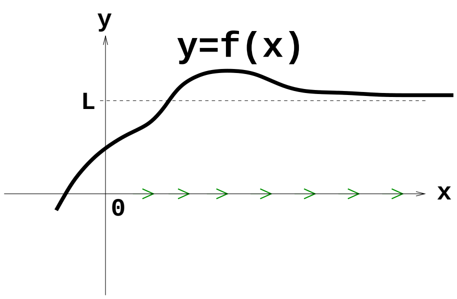


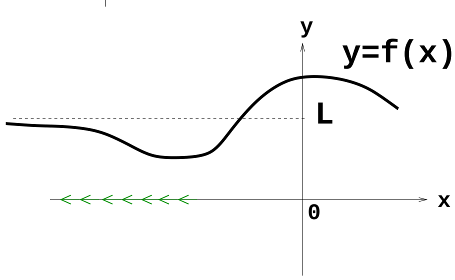
LIMIT COMPOSITION THEOREM: If f is continuous at b, where $\lim_{x\to a} g(x) = b$,

then $\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right) = f(b)$.

13 **Limits at Infinity**

(a) $\lim_{x\to\infty} f(x) = L$ (b) $\lim_{x\to-\infty} f(x) = L$





Remark: The line $y = L$ is a *Horizontal Asymptote* of $f(x)$.

14 Average rate of change of $y = f(x)$ over the interval $[x_1, x_2]$: $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$; slope of secant line through two points; average velocity. Definition of derivative of $y = f(x)$ at a: $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ or, equivalently, $f'(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x - a}$; interpretation of derivative:

$$f'(a) = \begin{cases} \text{slope of tangent line the graph of } y = f(x) \textit{ at } a \\ \text{velocity at time } a \\ \text{(instantaneous) rate of change of } f \text{ at } a \end{cases}$$

15 Derivative as a function: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \frac{dy}{dx}$; differentiable functions (i.e., $f'(x)$ exists); higher order derivatives : $f''(x) = \frac{d^2y}{dx^2}, \ldots$.

16 Average rate of change of $y = f(x)$ over the interval $[x_1, x_2]$: $\frac{\Delta y}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2 - x_1}$ (this is also the average velocity). Definition of derivative of $y = f(x)$ at a: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ or, equivalently, $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$; interpretation of derivative:

$$f'(a) = \begin{cases} \text{slope of tangent line the graph of } y = f(x) \text{ at } a \\ \text{velocity at time } a \\ \text{(instantaneous) rate of change of } f \text{ at } a \end{cases}$$

17 Derivative as a function: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$; differentiable functions (i.e., $f'(x)$ exists);

higher order derivatives: y'' or equivalently $\frac{d^2y}{dx^2}$; y''' or equivalently $\frac{d^3y}{dx^3}$; etc...

18 **Basic Differentiation Rules**: If f and g are differentiable functions, and c is a constant:

(a) $\frac{d(c)}{dx} = 0$ (b) $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ (c) $\frac{d(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

(c) `Power Rule`: $\frac{d(x^n)}{dx} = nx^{n-1}$

(d) `Product Rule` : $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ `Quotient Rule` : $\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

19 *Special Trig Limits* : $\boxed{\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$ $\boxed{\lim_{\theta\to 0} \frac{\theta}{\sin\theta} = 1}$ $\boxed{\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = 0}$.

Hence also $\lim_{\theta\to 0} \frac{\sin(k\theta)}{(k\theta)} = 1$ and $\lim_{\theta\to 0} \frac{(k\theta)}{\sin(k\theta)} = 1$. Note that $\sin k\theta \neq k\sin\theta$.

20 **CHAIN RULE**: If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $f \circ g$ is differentiable at x and its derivative is

$$(f\circ g)'(x) = \Big\{f(g(x))\Big\}' = f'(g(x))\, g'(x)$$

i.e., if $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

21 **Implicit Differentiation**: If an equation defines one variable as a function of the other (independent) variable, then to find the derivative of the function w.r.t. the independent variable:

`Step 1` - Differentiate both sides of equation w.r.t. independent variable

`Step 2` - Solve for the desired derivative

22 **Logarithmic Differentiation**

`Step 1` : Take natural log of both sides of $y = h(x)$; simplify using Law of Logarithms

`Step 2` : Differentiate implicitly w.r.t x

`Step 3` : Solve the resulting equation for $\dfrac{dy}{dx}$

23 **Inverse Trig Functions** - Note, for example, $\sin^{-1} x$ is same as $\arcsin x$, but $\sin^{-1} x \neq (\sin x)^{-1}$

(a) Definitions:

$$y = \sin^{-1} x \Longleftrightarrow \sin y = x \quad \left(-\tfrac{\pi}{2} \leq x \leq \tfrac{\pi}{2}\right)$$
$$y = \cos^{-1} x \Longleftrightarrow \cos y = x \quad (0 \leq x \leq \pi)$$
$$y = \tan^{-1} x \Longleftrightarrow \tan y = x \quad \left(-\tfrac{\pi}{2} < x < \tfrac{\pi}{2}\right)$$

(b) Basic Derivatives:

$$\boxed{\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}} \qquad \boxed{\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}} \qquad \boxed{\frac{d(\cos^{-1} x)}{dx} = -\frac{1}{\sqrt{1-x^2}}}$$

If u is a differentiable function of x, then by the Chain Rule, $\dfrac{d(\sin^{-1} u)}{dx} = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$, etc.

24 **Hyperbolic Trig Functions**

(a) Definitions:

$$\boxed{\cosh x = \frac{e^x + e^{-x}}{2}} \qquad \boxed{\sinh x = \frac{e^x - e^{-x}}{2}} \qquad \boxed{\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}}$$

(b) Basic Derivatives:

$$\boxed{\frac{d(\cosh x)}{dx} = \sinh x} \qquad \boxed{\frac{d(\sinh x)}{dx} = \cosh x} \qquad \boxed{\frac{d(\tanh x)}{dx} = \operatorname{sech}^2 x}$$

If u is a differentiable function of x, then by the Chain Rule, $\dfrac{d(\cosh u)}{dx} = (\sinh u)\dfrac{du}{dx}$, etc.

(c) Basic Identities:

$$\boxed{\cosh(-x) = \cosh x} \qquad \boxed{\sinh(-x) = -\sinh x} \qquad \boxed{\cosh^2 x - \sinh^2 x = 1}$$

$\boxed{25}$ APPLICATIONS

$\boxed{\texttt{Model 1}}$ - **Exponential Growth/Decay**: $\boxed{\dfrac{dy}{dt} = k\,y}$ where k = `relative growth/decay rate`

(If the rate of change of y is proportional to y, then the above differential equation holds.)

- If $k > 0$, this is the law of *Natural Growth* (for example, population growth).
- If $k < 0$, this is the law of *Natural Decay* (for example, radioactive decay).

All solutions to this differential equation have the form $\boxed{y(t) = y(0)\,e^{kt}}$.

(Usually need **two** pieces of information to determine both constants $y(0)$ and k, unless they are given explicitly.)

Half-life = time it takes for radioactive substance to lose half its mass.)

$\boxed{\texttt{Model 2}}$ - **Newton's Law of Cooling** : If $T(t)$ = temperature of an object at time t and T_s = temperature of its surrounding environment, then the rate of change of $T(t)$ is proportional to the difference between $T(t)$ and T_s :

$$\boxed{\frac{dT}{dt} = k\,(T(t) - T_s)}$$

The solution to this particular differential equation is always $\boxed{T(t) = T_s + Ce^{kt}}$

(Usually need **two** pieces of information to determine both constants C and k, unless they are given explicitly.)

$\boxed{\texttt{Model 3}}$ - **Related Rates** (Method to Solve) :

$\boxed{1}$ Read problem carefully several times to understand what is asked.

$\boxed{2}$ Draw a picture (if possible) and label.

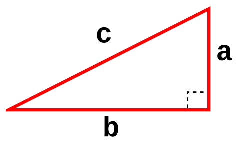
$\boxed{3}$ Write down the given rate; write down the desired rate.

$\boxed{4}$ Find an equation relating the variables.

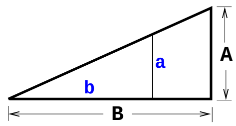
$\boxed{5}$ Use Chain Rule to differentiate equation w.r.t to time and solve for desired rate.

Useful Formulas for Related Rates

(i) **Pythagorean Theorem**: $c^2 = a^2 + b^2$

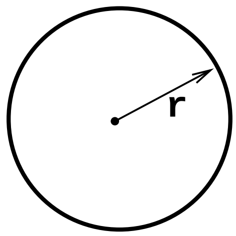


(ii) **Similar Triangles**: $\dfrac{a}{b} = \dfrac{A}{B}$



(iii) **Formulas from Geometry**:

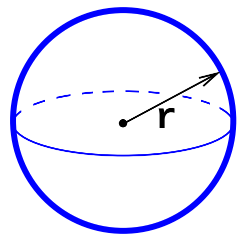
`Circle of radius` r



$A = \pi r^2$

$C = 2\pi r$

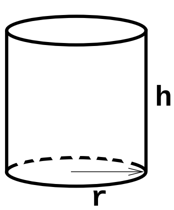
`Sphere of radius` r



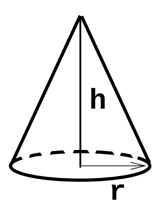
$V = \dfrac{4}{3}\pi r^3$

$S = 4\pi r^2$ (surface area of sphere)

`Cylinders and Cones:`



$V = \pi r^2 h$



$V = \dfrac{1}{3}\pi r^2 h$

Additional Differentiation Formulas

(u is a differentiable function of x)

$\dfrac{d\left(u^{n}\right)}{dx}=nu^{n-1}\dfrac{du}{dx}$	$\dfrac{d\left(e^{u}\right)}{dx}=e^{u}\dfrac{du}{dx}$	$\dfrac{d\left(a^{u}\right)}{dx}=a^{u}\left(\ln a\right)\dfrac{du}{dx}$

$\dfrac{d\left(\ln u\right)}{dx}=\dfrac{1}{u}\dfrac{du}{dx}$	$\dfrac{d\left(\log_a u\right)}{dx}=\dfrac{1}{u\ln a}\dfrac{du}{dx}$

$\dfrac{d\left(\sin u\right)}{dx}=\left(\cos u\right)\dfrac{du}{dx}$	$\dfrac{d\left(\cos u\right)}{dx}=\left(-\sin u\right)\dfrac{du}{dx}$	$\dfrac{d\left(\tan u\right)}{dx}=\left(\sec^2 u\right)\dfrac{du}{dx}$

$\dfrac{d\left(\csc u\right)}{dx}=\left(-\csc u\cot u\right)\dfrac{du}{dx}$	$\dfrac{d\left(\sec u\right)}{dx}=\left(\sec u\tan u\right)\dfrac{du}{dx}$	$\dfrac{d\left(\cot u\right)}{dx}=\left(-\csc^2 u\right)\dfrac{du}{dx}$

26 **Related Rates Word Problems Method**:

1. Read problem carefully several times to understand what is asked.
2. Draw a picture (if possible) and label.
3. Write down the given rate; write down the desired rate.
4. Find an equation relating the variables.
5. Use Chain Rule to differentiate equation w.r.t to time and solve for desired rate.

27 The *Linear Approximation* (or tangent line approximation) to a function $f(x)$ at $x=a$ is the function $L(x)=f(a)+f'(a)(x-a)$; Approximation formula $\boxed{f(x)\approx f(a)+f'(a)(x-a)}$ for x near a; if $y=f(x)$, the differential of y is $dy=f'(x)dx$.

28 Definitions of absolute maximum, absolute minimum, local/relative maxmium, and local/relative minimum; c is a *critical number* of f if c is in the domain of f and either $f'(c)=0$ or $f'(c)$ DNE.

29 **Extreme Value Theorem**: If $f(x)$ is continuous on a closed interval $[a,b]$, then f always has an absolute maximum value and an absolute minimum value on $[a,b]$.

30 **Method to Find Absolute Max/Min of $f(x)$ over Closed Interval $[a,b]$:**

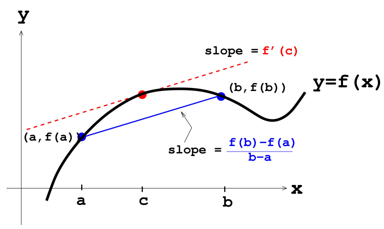
(i) Find all admissible critical numbers in (a,b);

(ii) Find endpoints of interval;

(iii) Make table of values of $f(x)$ at the points found in (i) and (ii).
The largest value = abs max value of f and the smallest value = abs min value of f.

31 **Rolle's Theorem**: If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), and $f(a) = f(b)$, then $f'(c) = 0$ for some $c \in (a,b)$.

32 **Mean Value Theorem**: If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there is a number c, where $a < c < b$, such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$:



i.e., $\boxed{f(b) - f(a) = f'(c)\,(b-a)}$

If something about f' is known, then something about the sizes of $f(a)$ and $f(b)$ can be found.

33 Fact: (Useful for integration theory later)

(a) If $f'(x) = 0$ for all $x \in I$, then $f(x) = C$ for all $x \in I$.

(b) If $f'(x) = g'(x)$ for all $x \in I$, then $f(x) = g(x) + C$ for all $x \in I$.

34 Increasing functions: $f'(x) > 0 \Longrightarrow f \nearrow$; decreasing functions: $f'(x) < 0 \Longrightarrow f \searrow$.

35 **First Derivative Test**: Suppose c is a critical number of a continuous function f.

(a) If f' changes from $+$ to $-$ at $c \Longrightarrow f$ has local max at c
(b) If f' changes from $-$ to $+$ at $c \Longrightarrow f$ has local min at c
(c) If f' does not change sign at $c \Longrightarrow f$ has neither local max nor local min at c

(*Displaying this information on a number line is much more efficient, see above figure.*)

36 f concave up: $f''(x) > 0 \Longrightarrow f \bigcup$; and f concave down: $f''(x) < 0 \Longrightarrow f \bigcap$;
inflection point (i.e. point where concavity changes).

(*Displaying this information on a number line is much more efficient, see above figure.*)

37 **Second Derivative Test**: Suppose f'' is continuous near critical number c and $f'(c) = 0$.

(a) If $f''(c) > 0 \Longrightarrow f$ has a local min at c.
(b) If $f''(c) < 0 \Longrightarrow f$ has a local max at c.

Note: If $f''(c) = 0$, then 2^{nd} Derivative Test cannot be used, so then use 1^{st} Derivative Test.

38 INDETERMINATE FORMS:

(a) **Indeterminate Form (Types)**: $\mathbf{\frac{0}{0}}$, $\mathbf{\frac{\infty}{\infty}}$, $\mathbf{0 \cdot \infty}$, $\mathbf{\infty - \infty}$, $\mathbf{0^0}$, $\mathbf{\infty^0}$, $\mathbf{1^\infty}$

(b) **L'Hopital's Rule**: Let f and g be differentiable and $g'(x) \neq 0$ on an open interval I containing a (except possibly at a). If $\lim_{x\to a} f(x) = 0$ and if $\lim_{x\to a} g(x) = 0$ [Type $\frac{0}{0}$]; or if $\lim_{x\to a} f(x) = \infty$ (or $-\infty$) and if $\lim_{x\to a} g(x) = \infty$ (or $-\infty$) [Type $\frac{\infty}{\infty}$], then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right side exists or is infinite.

Use algebra to convert the different Indetermine Forms in (a) *into expressions where the above formula can be used.*

Important Remark: L'Hopital's Rule is also valid for one-sided limits, $x \to a^-$, $x \to a^+$, and also for limits when $x \to \infty$ or $x \to -\infty$.

$\boxed{39}$ Curve Sketching Guidelines:

(a) `Domain` of f

(b) `Intercepts` (if any)

(c) `Symmetry`:
$f(-x) = f(x)$ for *even* function;
$f(-x) = -f(x)$ for *odd* function;
$f(x+p) = f(x)$ for *periodic* function

(d) `Asymptotes`:
$x = a$ is a *Vertical Asymptote*: if either $\lim_{x \to a^-} f(x)$ or $\lim_{x \to a^+} f(x)$ is infinite

$y = L$ is a *Horizontal Asymptote*: if either $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

(e) `Intervals`: where f is increasing $\nearrow$ and decreasing $\searrow$; local max and local min

(f) `Intervals`: where f is concave up $\bigcup$ and concave down $\bigcap$; inflection points

$\boxed{40}$ Optimization (Max/Min) Word Problems Method:

$\boxed{1}$ Read problem carefully several times.

$\boxed{2}$ Draw a picture (if possible) and label it.

$\boxed{3}$ Introduce notation for the quantity, say Q, to be extremized as a function of one or more variables.

$\boxed{4}$ Use information given in problem to express Q as a function of only one variable, say x. Write the domain of Q.

$\boxed{5}$ Use Max/Min methods to determine the absolute maximum value of Q or the absolute minimum of Q, whichever was asked for in problem.

$\boxed{41}$ Integration Theory:

(a) $F(x)$ is an *antiderivative* of $f(x)$, if $F'(x) = f(x)$.

(b) **Definite Integral** $\displaystyle\int_a^b f(x)\,dx$ is a number; gives the net area under a curve $y = f(x)$ when $a \le x \le b$; also gives net distance traveled by particle with velocity $y = f(x)$ from time $x = a$ to $x = b$; many other applications (take Calculus II).

(c) Properties of Definite Integrals.

(d) **FUNDAMENTAL THEOREM OF CALCULUS:**

$\boxed{\mathbf{I}}$ If $f(x)$ is continuous on $[a, b]$ and $g(x) = \displaystyle\int_a^x f(t)\,dt \implies g'(x) = f(x)$.

i.e., $\boxed{\dfrac{d}{dx}\left(\displaystyle\int_a^x f(t)\,dt\right) = f(x)}$ `FTC 1`

$\boxed{\textbf{II}}$ If $F(x)$ is any antiderivative of $f(x)$ $\Longrightarrow$ $\displaystyle\int_a^b f(x)\,dx = F(x)\Big|_{x=a}^{x=b} = F(b) - F(a)$.

i.e., $\boxed{\displaystyle\int_a^b F'(x)\,dx = F(b) - F(a)}$ FTC 2

(e) **Indefinite Integral** $\int f(x)\,dx$ is a function.

Recall that $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$, i.e. the indefinite integral $\int f(x)\,dx$ is simply the most general antiderivative of $f(x)$.

(f) `FTC 1 with Chain Rule:` $\boxed{\dfrac{d}{dx}\left(\displaystyle\int_a^{u(x)} f(t)\,dt\right) = f(u(x))\,\dfrac{du(x)}{dx}}$

(g) **Subsitution Rule** (Indefinite Integrals): $\boxed{\displaystyle\int f(g(x))\,g'(x)\,dx = \int f(u)\,du}$, $u = g(x)$.

(h) **Subsitution Rule** (Definite Integrals): $\boxed{\displaystyle\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$, $u = g(x)$.

Basic Table of Indefinite Integrals

(1) $\displaystyle\int k\,dx = kx + C$

(2) $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

(3) $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

(4) $\displaystyle\int e^x\,dx = e^x + C$

(5) $\displaystyle\int \cos x\,dx = \sin x + C$

(6) $\displaystyle\int \sin x\,dx = -\cos x + C$

$$(7) \quad \int \sec^2 x \, dx = \tan x + C$$

$$(8) \quad \int \sec x \, \tan x \, dx = \sec x + C$$

$$(9) \quad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$(10) \quad \int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C$$

$$(11) \quad \int \sinh x \, dx = \cosh x + C$$

$$(12) \quad \int \cosh x \, dx = \sinh x + C$$

$$(13) \quad \int a^x \, dx = \frac{a^x}{\ln a} + C$$