$$\lim_{x\to\infty} \frac{\sqrt{4x^2+2x}}{3x+1} =$$

A. ∞

B. $\frac{4}{3}$

C. 0

D. $\frac{2}{3}$

E. does not exist

limit at infinity: divide top and bottom by the highest degree of the denominator

here, divide by x

$$\lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+2x}}{x}}{\frac{3x+1}{x}} = \lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+2x}}{\sqrt{x^2}}}{3+\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{4x^2+2x}{x^2}}}{3+\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{4+\frac{2}{x}}}{3+\frac{1}{x}} = \frac{\sqrt{4}}{3} = \frac{2}{3}$$

$\frac{2}{x} \to 0$ $\qquad$ $\frac{1}{x} \to 0$

Choose the right statement which describes ALL the horizontal and vertical asymptotes of the function

$$f(x) = \frac{e^x + 1}{e^x - 1}$$

A. Horizontal Asymptote(s): $y = 1$, $y = -1$, Vertical Asymptote(s): None

B. Horizontal Asymptote(s): $y = 1$, Vertical Asymptote(s): $x = 1$

C. Horizontal Asymptote(s): $y = 1$, Vertical Asymptote(s): $x = 0$

D. Horizontal Asymptote(s): $y = 1$, $y = -1$, Vertical Asymptote(s): $x = 0$

E. Horizontal Asymptote(s): None, Vertical Asymptote(s): $x = 0$

vertical asymptote : when denom = 0 while numerator $\neq 0$

here, $e^x - 1 = 0$

$e^x = 1 \rightarrow \boxed{x = 0}$ one vertical asymptote at $x = 0$

horizontal asymptote to the right : $\lim_{x \to \infty}$

" " " left : $\lim_{x \to -\infty}$

right: $\lim_{x \to \infty} \frac{e^x + 1}{e^x - 1} \rightarrow \frac{e^{\text{big \#}} + 1}{e^{\text{big \#}} - 1}$ insignificant compared to e^x as $x \to \infty$

$\boxed{\lim_{x \to \infty} e^x = \infty}$ $\boxed{\lim_{x \to -\infty} e^x = 0}$

$= \frac{e^x}{e^x} = 1$ $\boxed{\text{horizontal asymptote (right): } y = 1}$

left: $\lim_{x \to -\infty} \frac{e^x + 1}{e^x - 1} = \frac{1}{-1} = -1$ $\boxed{\text{horizontal asymptote (left): } y = -1}$

$\rightarrow 0$

$\downarrow 0$

For what value(s) of c is

$$f(x) = \begin{cases} -cx+1, & \text{if } x < 2 \\ 3, & \text{if } x = 2 \\ c^2x^2+2, & \text{if } x > 2 \end{cases}$$

continuous from the left at 2.

Continuous from LEFT: $\lim_{x \to 2^-} f(x) = f(2)$

$$\underbrace{\lim_{x \to 2^-}}_{} f(x) = \lim_{x \to 2^-} (-cx+1) = -2c+1 = f(2) = 3$$

from left of 2

so $x < 2$

so top branch

Solve $-2c+1 = 3$

$-2c = 2$

$\boxed{c = -1}$

A. $-\frac{1}{2}\sqrt{\frac{3}{2}}, \frac{1}{2}\sqrt{\frac{3}{2}}$

B. 0

C. $\frac{1}{2}$

D. 2

E. -1

2. Let $f(x) = \dfrac{1}{\sqrt{x}}$. Which of the following equals $f'(4)$?

I. $\displaystyle\lim_{h\to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{\sqrt{4}}}{h}$ ✓

II. $\displaystyle\lim_{x\to 4} \frac{\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{x}}}{x-4}$ ✗

III. $\dfrac{-1}{16}$ ✓

limit definitions: $f'(a) = \displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ ①

$f'(a) = \displaystyle\lim_{x\to a} \frac{f(x) - f(a)}{x-a}$ ②

A. I. only

B. II. only

C. III. only

D. I. and III. only

E. I. and II. and III.

①: $f(x) = \dfrac{1}{\sqrt{x}}$

$f'(4) = \displaystyle\lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{4+h}} - \frac{1}{\sqrt{4}}}{h}$

②: $f(x) = \dfrac{1}{\sqrt{x}}$

$f'(4) = \displaystyle\lim_{x\to 4} \frac{f(x) - f(4)}{x-4} = \lim_{x\to 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{4}}}{x-4}$

III: $f(x) = \dfrac{1}{\sqrt{x}} = x^{-1/2}$ $\quad f'(x) = -\frac{1}{2}x^{-3/2} = \dfrac{-1}{2x^{3/2}}$

$f'(4) = -\frac{1}{2}\cdot\dfrac{1}{4^{3/2}} = -\frac{1}{2}\cdot\frac{1}{8} = -\frac{1}{16}$

Find the limit.

$$\lim_{x\to 0} \frac{\sin(4x)\sin(3x)}{x^2}$$

$$\boxed{\text{Special limit} \quad \lim_{x\to 0} \frac{\sin x}{x} = 1}$$

A. 0

B. 12

C. $\frac{1}{12}$

D. $\frac{3}{4}$

E. Does not exist.

$$\lim_{x\to 0} \frac{\sin(4x)}{x} \cdot \frac{\sin(3x)}{x}$$

want 4

want 3

so $\lim_{x\to 0} \frac{\sin(4x)}{4x} = 1$

$$= \lim_{x\to 0} \underbrace{\frac{\sin(4x)}{4x}}_{1} \cdot \underbrace{\frac{\sin(3x)}{3x}}_{1} \cdot \frac{4\cdot 3}{1} = 12$$

Suppose $f(1) = 4$ and $f'(1) = 3$. If

$$g(x) = \sqrt{f(x)}$$

then $g'(1)$ equals

$$g(x) = [f(x)]^{1/2}$$

treat it like u^n

deriv: $n u^{n-1} \cdot \frac{du}{dx}$

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. $\frac{1}{2\sqrt{3}}$

D. $\frac{2}{3}$

E. $\frac{1}{4}$

$$g'(x) = \frac{1}{2}[f(x)]^{-1/2} \cdot f'(x)$$

$$g'(x) = \frac{1}{2} \frac{1}{\sqrt{f(x)}} \cdot f'(x)$$

$$g'(1) = \frac{1}{2} \frac{1}{\sqrt{f(1)}} \cdot f'(1) = \frac{1}{2} \frac{1}{\sqrt{4}} \cdot 3 = \frac{3}{4}$$

The slope of the line tangent to the curve described by the implicit function $y^3x+y^2x^2=6$ at $(2,1)$ is

find $\frac{dy}{dx}$ at $x=2,\ y=1$

differentiate $y^3x+y^2x^2=6$ implicitly

$$\frac{d}{dx}(y^3x+y^2x^2)=\frac{d}{dx}(6)$$

A. $-\frac{3}{2}$

B. -1

C. $-\frac{3}{14}$

D. 0

E. $-\frac{5}{14}$

$$\underbrace{\frac{d}{dx}(y^3x)}_{\text{product rule}}+\underbrace{\frac{d}{dx}(y^2x^2)}_{\text{product rule}}=\frac{d}{dx}(6)$$

$$\left[(y^3)(1)+(x)(3y^2)\frac{dy}{dx}\right]+\left[(y^2)(2x)+(x^2)(2y)\frac{dy}{dx}\right]=0$$

plug in $x=2$ $y=1$

$$\left(1+6\frac{dy}{dx}\right)+\left(4+8\frac{dy}{dx}\right)=0$$

$$14\frac{dy}{dx}=-5 \qquad \frac{dy}{dx}=\boxed{-\frac{5}{14}}$$

F21 exam 2

#2. $y = \dfrac{t^2 e^{-t}}{t^{1/2}+1}$

find $\dfrac{dy}{dt}$ at $t=1$

quotient to start

product rule

$$\frac{d}{dt}(t^2 e^{-t}) = t^2 \cdot e^{-t} \cdot -1 + e^{-t} \cdot 2t$$

$$y' = \frac{(t^{1/2}+1)\boxed{\frac{d}{dt}(t^2 e^{-t})} - (t^2 e^{-t})(\frac{1}{2}t^{-1/2})}{(t^{1/2}+1)^2}$$

$$= \frac{(t^{1/2}+1)(-t^2 e^{-t} + 2te^{-t}) - (t^2 e^{-t})(\frac{1}{2}t^{-1/2})}{(t^{1/2}+1)^2}$$

plug in $t=1$

$$= \frac{(2)(-e^{-t}+2e^{-1}) - (e^{-1})(\frac{1}{2})}{4} = \frac{+2e^{-1} - \frac{1}{2}e^{-1}}{4} = \cancel{\frac{-\frac{5}{2}e^{-1}}{4}} \cancel{= -\frac{5}{8}e^{-1}} = -\frac{5}{8e}$$

$$\frac{\frac{3}{2}e^{-1}}{4} = \boxed{\frac{3}{8e}}$$

F21 exam 2

#8 $f(x) = \cos(\pi e^{3x})$

$\frac{d}{dx}\cos(u) = -\sin(u)\frac{du}{dx}$

find $f'(\frac{1}{3}\ln\frac{1}{2})$

$f'(x) = -\sin(\pi e^{3x}) \cdot \frac{d}{dx}(\pi e^{3x})$

$\frac{d}{dx}e^{u} = e^{u}\frac{du}{dx}$

$= -\sin(\pi e^{3x}) \cdot \pi e^{3x} \cdot 3$

$= e^{\ln\frac{1}{2}} = \frac{1}{2}$

$f'(\frac{1}{3}\ln\frac{1}{2}) = -\sin(\pi e^{3\cdot\frac{1}{3}\ln\frac{1}{2}}) \cdot \pi e^{3\cdot\frac{1}{3}\ln\frac{1}{2}} \cdot 3$

$= \underbrace{-\sin(\frac{\pi}{2})}_{-1} \cdot 3\pi \cdot \frac{1}{2} = -1 \cdot \frac{3\pi}{2} = -\frac{3\pi}{2}$